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KIEPERT CIRCLES, SEMI KIEPERT CIRCLES AND EULER LINE OF TRIANGLE

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ABSTRACT. Firstly we will introduce the concept of Kiepert circles, semi Kiepert circles of triangle. Then, we will prove that Kiepert circles take the Euler line of triangle to be their common radical axis and semi Kiepert circles also take the Euler line of triangle to be their common radical axis.

I. Introduction.

Let be non-isosceles triangle ABC and points X, Y, Z such that triangles XBC, YCA, ZAB are isosceles at X, Y, Z respectively and these triangles are similar in the same direction. Let A_0, B_0, C_0 be the midpoints of BC, CA, AB. Let A_1, B_1, C_1 be the intersection points of B_0C_0, C_0A_0, A_0B_0 with YZ, ZX, XY respectively. Let A_2, B_2, C_2 be the intersection points of B_0C_0, C_0A_0, A_0B_0 with YZ, ZX, XY respectively. The circles with diameters AA_1, BB_1, CC_1 are called Kiepert circles respectively corresponding to vertices A, B, C of triangles. The circles with diameters A_0A_2, B_0B_2, C_0C_2 are together called semi Kiepert circles respectively correspongding to A, B, C of triangle ABC. Without the need to emphasize, the circles with diameters AA_1, BB_1, CC_1 are together called Kiepert circles of triangle ABC. The circles with diameters A_0A_2, B_0B_2, C_0C_2 are together called semi Kiepert circles of triangle ABC.
We can find the meaning of the term of Kiepert and semi Kiepert circles in [1].
In this paper we will prove that Kiepert circles of non-isosceles triangle ABC take the Euler line of this triangle ABC to be their common radical axis and semi Kiepert circles of non-isosceles triangle ABC also take the Euler line of this triangle to be their common radical axis.
Let $BC = a, AC = b, AB = c$. Let R be the circumradius of triangle ABC. Let $P(M/(O))$ be the power of point M with respect to circle (O).
For the reader's convenience, we will introduce some other notations:
The outer product of two vectors $\vec{a}$ and $\vec{b}$ is $\vec{a} \wedge \vec{b}$.
The signed area of triangle ABC is $S[ABC]$.
Vector rotation with angle α is R^{α}.
We can find basic knowledge on the outer product and signed area in [2].

II. Lemmas.

Lemma 1. Let be triangle ABC and points X, Y, Z such that triangles XBC, YCA, ZAB are isosceles at X, Y, Z respectively and these triangles are similar in the same direction. Let B_0, C_0 be the midpoints of AC, AB respectively. Then,
1. Triangles ABC, XYZ have a common centroid.

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2. $S[XB_0Y] = -S[XC_0Z]$.
Proof.

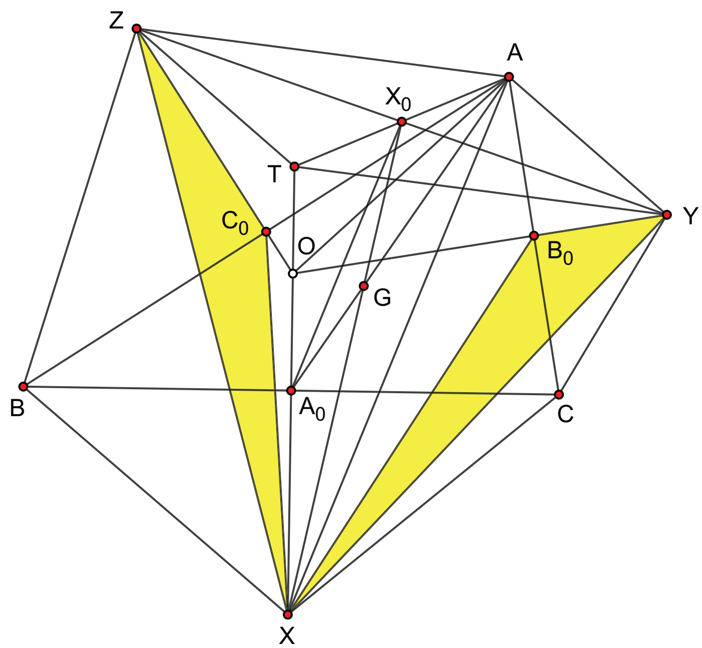

Figure 1

1. Let A_0 be the midpoint of BC (figure 1). Suppose that triangles XBC, YCA, ZAB are similar in the same negative direction and isosceles at X, Y, Z respectively , we have
$(\overrightarrow{BX}; \overrightarrow{BC}) = (\overrightarrow{CY}; \overrightarrow{CA}) = (\overrightarrow{AZ}; \overrightarrow{AB}) = \alpha (mod 2\pi)$ with $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$.
Hence, $\overrightarrow{BX} + \overrightarrow{CY} + \overrightarrow{AZ} = \frac{1}{cos\alpha} R^{\alpha}(\overrightarrow{BA_0}) + \frac{1}{cos\alpha} R^{\alpha}(\overrightarrow{CB_0}) + \frac{1}{cos\alpha} R^{\alpha}(\overrightarrow{AC_0}) = \frac{1}{cos\alpha} R^{\alpha}(\overrightarrow{BA_0} + \overrightarrow{CB_0} + \overrightarrow{AC_0}) = \frac{1}{2cos\alpha} R^{\alpha}(\overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB}) = \vec{0}$.
This means that triangles ABC, XYZ have a common centroid.
2. Let G be the common centroid of triangles ABC, XYZ. Let X_0 be the midpoint of YZ. Let T be the intersection point of XA_0 with AX_0. Let O be the circumcenter of triangle ABC (figure 1).
We see that $G = AA_0 \cap XX_0$ and $\frac{\overline{GA_0}}{\overline{GA}} = \frac{\overline{GX_0}}{\overline{GX}} = -\frac{1}{2}$. Hence, by Thales's theorem, $A_0X_0 \parallel AX$ and $\frac{\overline{A_0X_0}}{\overline{AX}} = -\frac{1}{2}$. It follows that, by Thales' theorem, $\frac{\overline{TX_0}}{\overline{TA}} = \frac{\overline{TA_0}}{\overline{TX}} = \frac{\overline{X_0A_0}}{\overline{AX}} = \frac{1}{2}$. This means that X_0, A_0 are midpoints of AT, XT respectively. So,
$S[XB_0Y] + S[XC_0Z] = S[B_0YX] + S[C_0ZX] = \frac{1}{2}\overrightarrow{B_0Y} \wedge \overrightarrow{B_0X} + \frac{1}{2}\overrightarrow{C_0Z} \wedge \overrightarrow{C_0X}$
$= \frac{1}{2}\overrightarrow{B_0Y} \wedge (\overrightarrow{OX} - \overrightarrow{OB_0}) + \frac{1}{2}\overrightarrow{C_0Z} \wedge (\overrightarrow{OX} - \overrightarrow{OC_0}) = \frac{1}{2}(\overrightarrow{B_0Y} + \overrightarrow{C_0Z}) \wedge \overrightarrow{OX} = \frac{1}{2}(\frac{1}{2}(\overrightarrow{AY} + \overrightarrow{CY}) + \frac{1}{2}(\overrightarrow{AZ} + \overrightarrow{BZ})) \wedge \overrightarrow{OX} = \frac{1}{4}((\overrightarrow{AY} + \overrightarrow{AZ}) + (\overrightarrow{BZ} + \overrightarrow{CY})) \wedge \overrightarrow{OX} = \frac{1}{4}(\overrightarrow{AT} + 2\overrightarrow{A_0X_0}) \wedge$

$\overrightarrow{OX} = \frac{1}{4}.2.(\overrightarrow{A_0X_0} + \overrightarrow{X_0T}) \wedge \overrightarrow{OX} = \frac{1}{2}\overrightarrow{A_0T} \wedge \overrightarrow{OX} = 0$. This means that $S[XB_0Y] = -S[XC_0Z]$.

Lemma 2. Let be triangle ABC and points X, Y, Z such that triangles XBC, YCA, ZAB are isosceles at X, Y, Z respectively and these are similar in the same direction. Let A_0, B_0, C_0 be the midpoints of BC, CA, AB respectively. Let B_1, C_1 be the intersection points of CA, AB with ZX, XY respectively. Let B_2, C_2 be the intersection points of C_0A_0, A_0B_0 and ZX, XY respectively. Then, $\frac{\overline{BC_1}}{\overline{CB_1}} = \frac{\overline{B_0C_2}}{\overline{C_0B_2}}$.

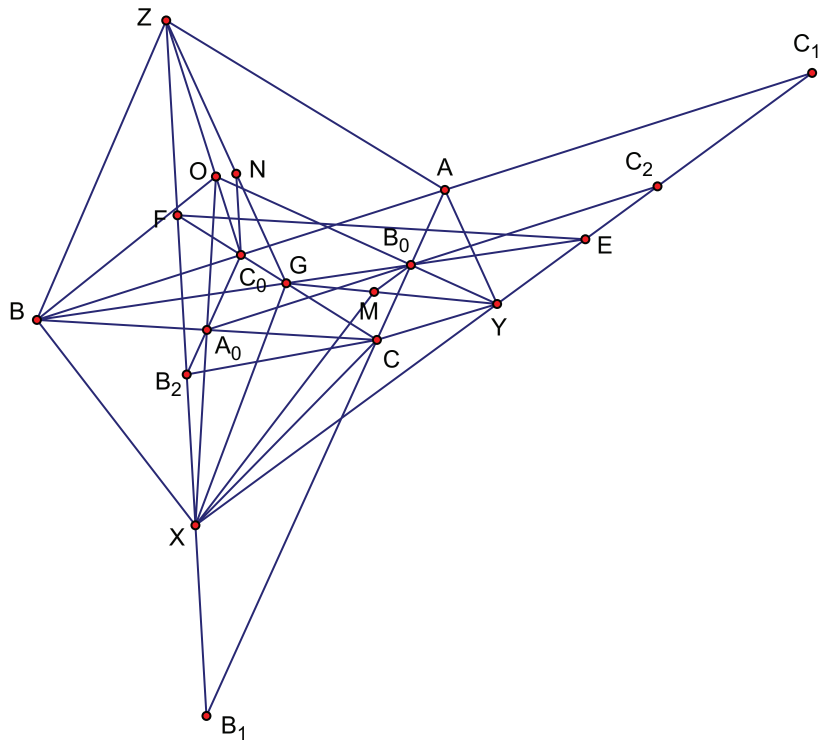

Figure 2

Proof. Let G be the common centroid of triangles ABC, XYZ (see lemma 1). Let E, F be the intersection points of BB_0, CC_0 with C_1C_2, B_1B_2 respectively. Let M, N be the intersection points of GY, GZ with the lines passing through B_0, C_0 and parallel to XY, XZ respectively (figure 2). Since G is the centroid of triangle XYZ, $S[XGY] = \frac{1}{3}S[XZY] = -\frac{1}{3}S[XYZ] = -S[XGZ]$. By lemma 1, $S[XB_0Y] = -S[XC_0Z]$. From this, noting that $B_0M \parallel EY; C_0N \parallel FZ$, by Thales's theorem, we obtain $\frac{\overline{GE}}{\overline{B_0E}} = \frac{\overline{GY}}{\overline{MY}} = \frac{S[XGY]}{S[XMY]} = \frac{S[XGY]}{S[XB_0Y]} = \frac{S[XGZ]}{S[XC_0Z]} = \frac{S[XGZ]}{S[XNZ]} = \frac{\overline{GZ}}{\overline{NZ}} = \frac{\overline{GF}}{\overline{C_0F}}$.

Thus, noting that $G = BB_0 \cap CC_0$ and $\frac{\overline{GB_0}}{\overline{GB}} = \frac{\overline{GC_0}}{\overline{GC}} = -\frac{1}{2}$, we get $\frac{\overline{BE}}{\overline{B_0E}} = \frac{\overline{CF}}{\overline{C_0F}}$.

Combining with $BC_1 \parallel B_0C_2; CB_1 \parallel C_0B_2$, by Thales's theorem, we have $\frac{\overline{BC_1}}{\overline{B_0C_2}} = \frac{\overline{CB_1}}{\overline{C_0B_2}}$. This means that $\frac{\overline{BC_1}}{\overline{CB_1}} = \frac{\overline{B_0C_2}}{\overline{C_0B_2}}$.

Lemma 3. Let be triangle ABC and points X, Y, Z such that triangles XBC, YCA, ZAB are isosceles at X, Y, Z respectively and these are similar in the same direction. Let O be the circumcircle of triangle ABC. Let A_0, B_0, C_0 be the midpoints of BC, CA, AB respectively. Let B_2, C_2 be the intersection points of C_0A_0, A_0B_0 with ZX, XY respectively. Let B_3, C_3 be the intersection points of CA, AB with the lines passing through O and perpendicular to OB, OC respectively. Then, $\frac{\overline{B_0C_2}}{\overline{C_0B_2}} = \frac{\overline{BC_3}}{\overline{CB_3}}$.

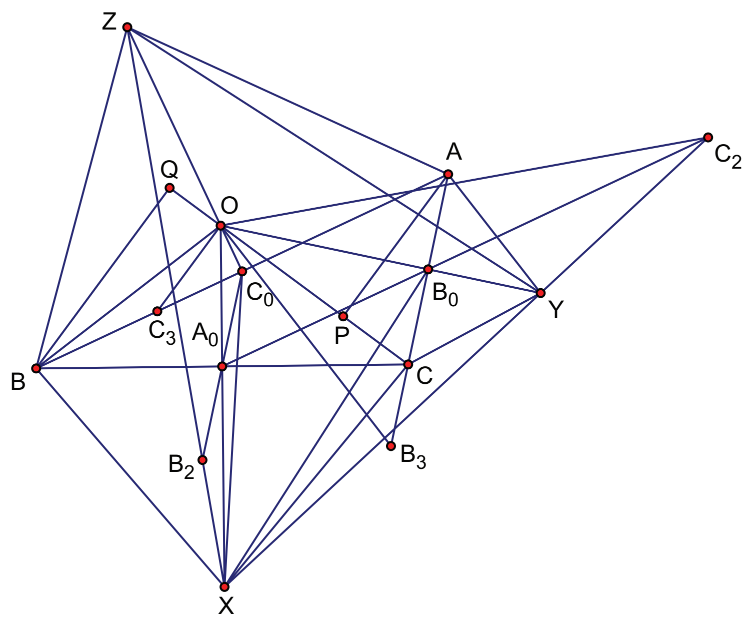

Figure 3

Proof. Without the loss of generality, suppose that triangles XBC, YCA, ZAB are in the negative direction. Because triangles XBC, YCA, ZAB are isosceles at X, Y, Z respectively and these triangles are similar in the same direction, there exists $\alpha \in (0; \frac{\pi}{2})$ such that $\angle XBC = \angle YCA = \angle ZAB = \alpha$.

Let P, Q be the projection points of A, B onto OC respectively (figure 3).

Because C_2, X, Y are collinear, applying Menelaus's theorem to triangle OA_0B_0, we have $\frac{\overline{C_2A_0}}{\overline{C_2B_0}}.\frac{\overline{YB_0}}{\overline{YO}}.\frac{\overline{XO}}{\overline{XA_0}} = 1$.

Because $C_3O \perp OC; AP \perp OC; BQ \perp OC$, so $C_3O \parallel AP \parallel BQ$. Hence, by Thales's theorem, we have

$$\frac{\overline{C_3A}}{\overline{C_3B}} = \frac{\overline{OP}}{\overline{OQ}} = \frac{\overline{OP}.\overline{OC}}{\overline{OQ}.\overline{OC}} = \frac{2.\overrightarrow{OA}.\overrightarrow{OC}}{2.\overrightarrow{OB}.\overrightarrow{OC}} = \frac{OA^2 + OC^2 - AC^2}{OB^2 + OC^2 - BC^2} = \frac{2R^2 - b^2}{2R^2 - a^2}.$$

Because B_0, A_0 are the midpoints of CA, CB respectively, $2\overrightarrow{CB_0} = \overrightarrow{CA}; 2\overrightarrow{CA_0} = \overrightarrow{CB}$.

Because triangles YCA, XBC are similar in the same negative direction and these triangles are isosceles at Y, X respectively, triangles CYA, BXC are similar in the same positive direction; $CY = \dfrac{CA}{2cos\alpha}; CX = \dfrac{CB}{2cos\alpha}; (\overrightarrow{CY};\overrightarrow{CB}) + (\overrightarrow{CX};\overrightarrow{CA}) \equiv (\overrightarrow{CY};\overrightarrow{CA}) + (\overrightarrow{CX};\overrightarrow{CB}) = (\overrightarrow{CY};\overrightarrow{CA}) - (\overrightarrow{BX};\overrightarrow{BC}) \equiv 0 (mod 2\pi)$.

Hence, $S[CYB] + S[CXA] = \frac{1}{2}.CY.CB.sin(\overrightarrow{CY};\overrightarrow{CB}) + \frac{1}{2}.CX.CA.sin(\overrightarrow{CX};\overrightarrow{CA}) = \dfrac{CA.CB}{4cos\alpha}(sin(\overrightarrow{CY};\overrightarrow{CB}) + sin(\overrightarrow{CX};\overrightarrow{CA})) = 0$; $S[CYA] - S[BXC] = \frac{1}{2}.CY.CA.sin\angle YCA - \frac{1}{2}.BX.BC.sin\angle XBC = \dfrac{CA^2}{4cos\alpha}.sin\alpha - \dfrac{BC^2}{4cos\alpha}.sin\alpha = \frac{1}{4}tan\alpha.(b^2 - a^2)$.

Therefore,

$$2.\frac{\overline{B_0C_2}}{\overline{BC_3}} = \frac{\overline{B_0C_2}}{\overline{A_0B_0}}.\frac{\overline{BA}}{\overline{BC_3}} = \frac{\overline{C_2B_0}}{\overline{C_2B_0} - \overline{C_2A_0}}.\frac{\overline{C_3A} - \overline{C_3B}}{\overline{C_3B}} =$$

$$\frac{1}{1 - \dfrac{\overline{C_2A_0}}{\overline{C_2B_0}}}.(\frac{\overline{C_3A}}{\overline{C_3B}} - 1) = \frac{1}{1 - \dfrac{\overline{YO}}{\overline{YB_0}}.\dfrac{\overline{XA_0}}{\overline{XO}}}(\frac{2R^2 - b^2}{2R^2 - a^2} - 1) = \frac{1}{1 - \dfrac{S[XYO]}{S[XYB_0]}.\dfrac{S[YXA_0]}{S[YXO]}}.\frac{a^2 - b^2}{2R^2 - a^2}$$

$$= \frac{1}{1 - \dfrac{S[XYO]}{S[XYB_0]}.\dfrac{S[XYA_0]}{S[XYO]}}.\frac{a^2 - b^2}{2R^2 - a^2}$$

$$= \frac{S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{S[XYB_0] - S[XYA_0]} = \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{2S[XYB_0] - 2S[XYA_0]}$$

$$= \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{\overrightarrow{XY} \wedge \overrightarrow{XB_0} - \overrightarrow{XY} \wedge \overrightarrow{XA_0}}$$

$$= \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{\overrightarrow{XY} \wedge (\overrightarrow{XB_0} - \overrightarrow{XA_0})} = \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{(\overrightarrow{CY} - \overrightarrow{CX}) \wedge (\overrightarrow{CB_0} - \overrightarrow{CA_0})}$$

$$= \frac{4S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{(\overrightarrow{CY} - \overrightarrow{CX}) \wedge (2\overrightarrow{CB_0} - 2\overrightarrow{CA_0})} = \frac{4S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{(\overrightarrow{CY} - \overrightarrow{CX}) \wedge (\overrightarrow{CA} - \overrightarrow{CB})}$$

$$= \frac{4S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{\overrightarrow{CY} \wedge \overrightarrow{CA} - \overrightarrow{CY} \wedge \overrightarrow{CB} - \overrightarrow{CX} \wedge \overrightarrow{CA} + \overrightarrow{CX} \wedge \overrightarrow{CB}}$$

$$= \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{S[CYA] - S[CYB] - S[CXA] + S[CXB]} = \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{S[CYA] + S[CXB]}$$

$$= \frac{2S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{S[CYA] - S[BXC]} = \frac{8S[XYB_0]}{2R^2 - a^2}.\frac{a^2 - b^2}{tan\alpha.(b^2 - a^2)} = \frac{8S[XB_0Y]}{(2R^2 - a^2)tan\alpha}.$$

Similarly, noting that $S[BZA] - S[CXB] = -\frac{1}{4}tan\alpha(c^2 - a^2)$, we have $2.\dfrac{\overline{C_0B_2}}{\overline{CB_3}} = \dfrac{-8S[XC_0Z]}{(2R^2 - a^2).tan\alpha}$.

Thus, by lemma 1, we have $\dfrac{\overline{B_0C_2}}{\overline{BC_3}} = \dfrac{\overline{C_0B_2}}{\overline{CB_3}}$. This means that $\dfrac{\overline{B_0C_2}}{\overline{C_0B_2}} = \dfrac{\overline{BC_3}}{\overline{CB_3}}$.

III.Theorems.

Theorem 1. Kiepert circles of non-isosceles triangle ABC take the Euler line of this triangle to be their common radical axis.

Proof.
Let be points X, Y, Z such that triangles XBC, YCA, ZAB are isosceles at X, Y, Z respectively and these are similar in the same direction.
Let A_1, B_1, C_1 be the intersection points of BC, CA, AB with lines YZ, ZX, XY respectively.

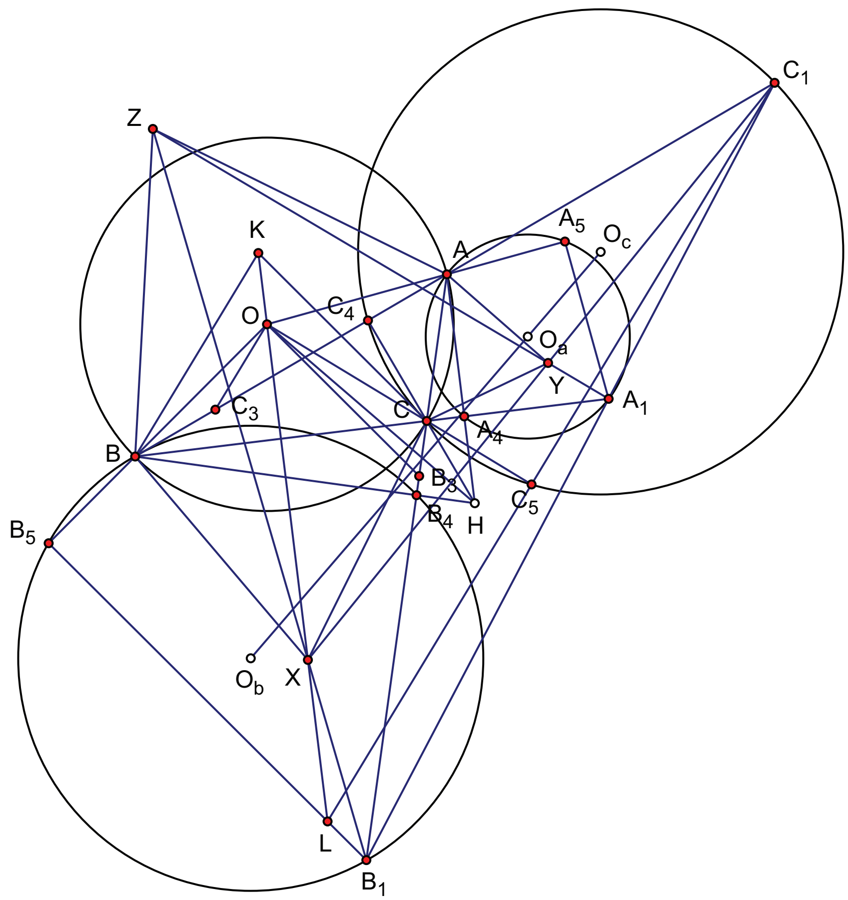

Figure 4

Let O, H be the circumcenter and orthocenter of triangle ABC respectively. Let B_3, C_3 be the intersection points of the lines passing through O and perpendicular to OB, OC with CA, AB respectively. Let A_4, B_4, C_4 be the intersection points of AH, BH, CH with BC, CA, AB respectively. Let B_5, C_5 be the projection points of B_1, C_1 onto OB, OC respectively. Let K be the intersection point of line passing through B and perpendicular to OC with line passing through C and perpendicular OB. Let L be the intersection point of B_1B_5 with C_1C_5. Let $(O_a), (O_b), (O_c)$ be the circles with diameters AA_1, BB_1, CC_1 respectively (figure 4).

It is easy to see that $\overline{HA}.\overline{HA_4} = \overline{HB}.\overline{HB_4} = \overline{HC}.\overline{HC_4}$. From this, noting that A_4, B_4, C_4 lie on $(O_a), (O_b), (O_c)$ respectively, we have $P(H/(O_a)) = P(H/(O_b)) = P(H/(O_c))(1)$.

Moreover, by lemmas 2 and 3, we obtain $\dfrac{\overline{BC_1}}{\overline{CB_1}} = \dfrac{\overline{BC_3}}{\overline{CB_3}}$. This means that $\dfrac{\overline{BC_1}}{\overline{BC_3}} = \dfrac{\overline{CB_1}}{\overline{CB_3}}$.

Combining with $BK \parallel C_3O \parallel C_1C_5; CK \parallel B_3O \parallel B_1B_5$, by Thales's theorem, we deduce that K, O, L are collinear. Combining with $KB = KC$, it follows that triangles OBL and OCL

are equal.
Combining with $LB_5 \perp OB; LC_5 \perp OC$, noting that B_5, C_5 lie on $(O_b), (O_c)$ respectively, we obtain $P(O/(O_b)) = \overline{OB}.\overline{OB_5} = \overline{OC}.\overline{OC_5} = P(O/(O_c))$.
Similarly, $P(O/(O_a)) = P(O/(O_b)) = P(O/(O_c))(2)$.
From (1) and (2), the proof is done.

Theorem 2. Semi Kiepert circles of non-isosceles triangle ABC take the Euler line of this triangle to be their common radical axis.

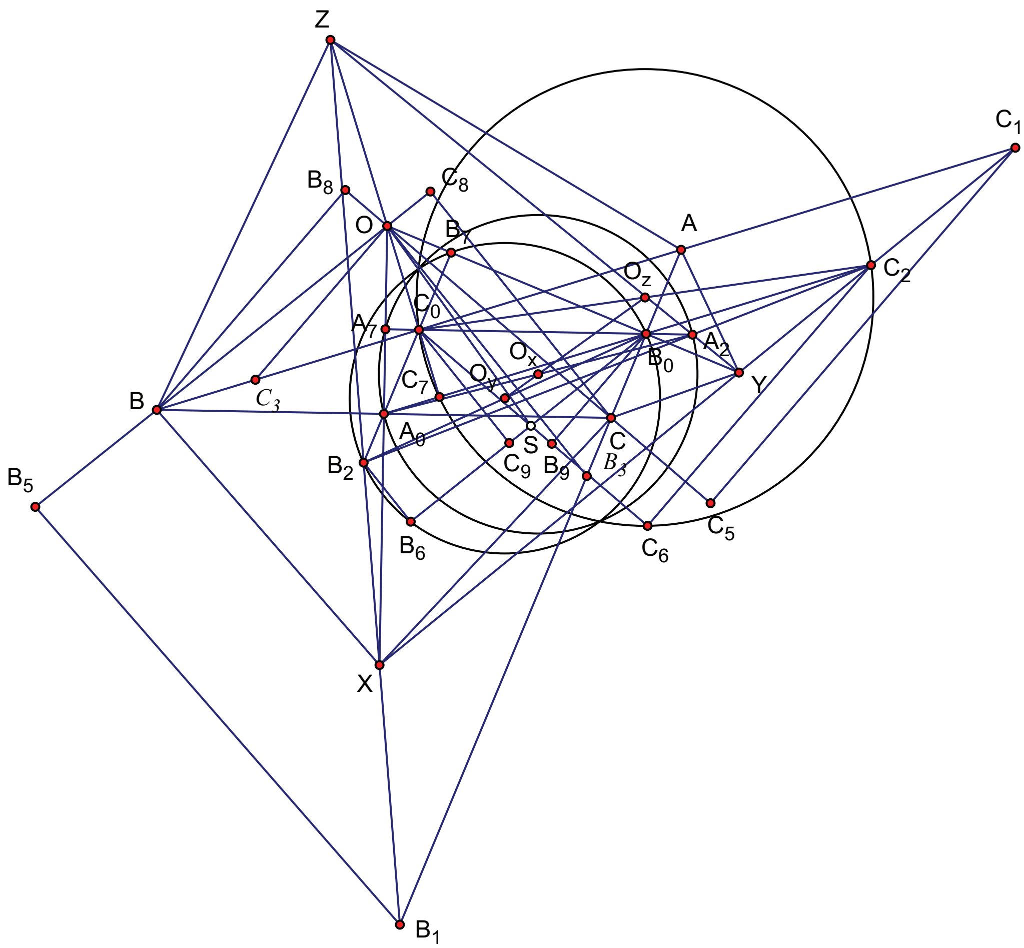

Figure 5

Proof.

Let be points X, Y, Z such that triangles XBC, YCA, ZAB are isosceles at X, Y, Z and these are similar in the same direction.

Let A_0, B_0, C_0 be the midpoints of BC, CA, AB respectively. Let A_2, B_2, C_2 be the intersection points of B_0C_0, C_0A_0, A_0B_0 with YZ, ZX, XY respectively. Let O, S be the circumcircle of triangles $ABC, A_0B_0C_0$ respectively. Let B_1, C_1 be the intersection points of CA, AB with ZX, XY respectively. Let B_5, C_5 be the projection points of B_1, C_1 onto OB, OC respectively. Let B_6, C_6 be the projection points of B_2, C_2 onto SB_0, SC_0 respectively. Let A_7, B_7, C_7 be the intersection points A_0O, B_0O, C_0O with B_0C_0, C_0A_0, A_0B_0 respectively. Let B_8, C_8 be the projection points of B, C onto OC, OB. Let B_9, C_9 be the projection points of B_0, C_0 onto SC_0, SB_0 respectively. Let $(O_x), (O_y), (O_z)$ be the circles with diameters A_0A_2, B_0B_2, C_0C_2 respectively (figure 5).

It is easy to see that O is the orthocenter of triangle $A_0B_0C_0$. Hence, $\overline{OA_0}.\overline{OA_7} = \overline{OB_0}.\overline{OB_7} = \overline{OC_0}.\overline{OC_7}$.

From this, noting that A_7, B_7, C_7 lie on $(O_x), (O_y), (O_z)$ respectively, we have $P(O/(O_x)) = P(O/(O_y)) = P(O/(O_z))(3)$.

By lemma 2, $\dfrac{\overline{CB_1}}{\overline{C_0B_2}} = \dfrac{\overline{BC_1}}{\overline{B_0C_2}}$. It is easy to see that $CC_8 \parallel B_1B_5 \parallel C_0C_9 \parallel B_2B_6; OB \parallel B_0S; BB_8 \parallel C_1C_5 \parallel B_0B_9 \parallel C_2C_6; OC \parallel C_0S$. So, nothing that parallel projection preserves ratio of signed lengths of two directed segments belonging to parallel lines, we get $\dfrac{\overline{C_8B_5}}{\overline{C_9B_6}} = \dfrac{\overline{B_8C_5}}{\overline{B_9C_6}}$.

This means that $\dfrac{\overline{B_8C_5}}{\overline{C_8B_5}} = \dfrac{\overline{B_9C_6}}{\overline{C_9B_6}}(4)$.

As the proof of theorem 1, we have $\overline{OB}.\overline{OB_5} = \overline{OC}.\overline{OC_5}$.

It is easy to see that quadrilateral BCC_8B_8 is inscribed. Hence, $\overline{OB}.\overline{OB_8} = \overline{OC}.\overline{OC_8}$.

So, noting that $-2\overline{SB_0} = \overline{OB}; -2\overline{SC_0} = \overline{OC}$, we get $-2\overline{SB_0}.\overline{C_8B_5} = \overline{OB}.(\overline{OB_5} - \overline{OC_8}) = \overline{OB}.\overline{OB_5} - \overline{OB}.\overline{OC_8} = \overline{OC}.\overline{OC_5} - \overline{OC}.\overline{OB_8} = \overline{OC}(\overline{OC_5} - \overline{OB_8}) = -2\overline{SC_0}.\overline{B_8C_5}$ (5).

From (4) and (5), we have $\dfrac{\overline{SB_0}}{\overline{SC_0}} = \dfrac{\overline{B_9C_6}}{\overline{C_9B_6}}$.

Hence, $\overline{SB_0}.\overline{SB_6} - \overline{SB_0}.\overline{SC_9} = \overline{SB_0}.(\overline{SB_6} - \overline{SC_9}) = \overline{SB_0}.\overline{C_9B_6}$
$= \overline{SC_0}.\overline{B_9C_6} = \overline{SC_0}.(\overline{SC_6} - \overline{SB_9}) = \overline{SC_0}.\overline{SC_6} - \overline{SC_0}.\overline{SB_9}$ (6).

It is easy to see that $B_0C_0C_9B_9$ is inscribed, so $\overline{SB_0}.\overline{SC_9} = \overline{SC_0}.\overline{SB_9}$ (7).

From (6) and (7) we have $P(S/(O_y)) = \overline{SB_0}.\overline{SB_6} = \overline{SC_0}.\overline{SC_6} = P(S/(O_z))$.

Similarly, $P(S/(O_x)) = P(S/(O_y)) = P(S/(O_z))$ (8).

From (1) and (8), noting that S is Euler center of triangle ABC, the proof is done.

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