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# RESULTS CONCERNING THE ANALYSIS OF INTEGRAL TRANSFORMS

N.U. KHAN, T. USMAN AND M.GHAYASUDDIN

ABSTRACT. An attempt has been made in this paper to unify and extend the result by establishing an integral transform involving Bessel function $J_\nu(z)$ into a multiple hypergeometric series of Lauricella function $F_A^{(n)}$. Certain special cases of the main result, which are of interest in themselves and do not seem to be recorded in the literature have also been given.

## 1. Introduction and Definition

Integral transformations have been successfully used for almost two centuries in solving many problems in applied mathematics, mathematical physics and engineering sciences. The origin of integral transforms including the Laplace and Fourier transforms can be traced back to celebrated work on probability theory. Many authors have discussed various transformations and interesting instances of the reducibility of triple hypergeometric functions. Of the various methods available for obtaining transformation on multiple hypergeometric function, the manipulation of integral representing the function may often be employed a good effect.

A Lauricella function of $n$ variables $F_A^{(n)}$ is defined as follows (see [4],[8; p. 60]).

$$F_A^{(n)}[a, b_1, \ldots, b_n; c_1, \ldots, c_n; x_1, \ldots, x_n] = \sum_{m_1,\ldots,m_n=0}^{\infty} \frac{(a)_{m_1+\cdots+m_n}(b_1)_{m_1}\ldots(b_n)_{m_n}}{(c_1)_{m_1}\ldots(c_n)_{m_n}} \frac{x_1^{m_1}}{m_1!}\cdots\frac{x_n^{m_n}}{m_n!}, \tag{1.1}$$

$$| x_1 | + \cdots + | x_n | < 1 .$$

Some transformations of Lauricella function are given below [8; p. 61 (5),(6)],

$$F_A^{(1)} = {}_2F_1,\ F_A^{(2)} = F_2 \tag{1.2}$$

where ${}_2F_1, F_2$ are Gauss hypergeometric function and Appell function.

In 1921, Appell's four hypergeometric functions $F_1, F_2, F_3, F_4$ and seven his confluent

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forms $\phi_1, \phi_2, \phi_3, \psi_1, \psi_2, \Xi_1, \Xi_2$ were unified and generalized by Kampé de Fériet. We recall here the definition of a more general double hypergeometric function of Kampé de Fériet in a slightly modified notation of Shivastava and Panda (see [10]):

$$F^{A:\,B;\,D}_{E:\,G;\,H}\left[\begin{matrix}(a_A): & (b_B); & (d_D); \\ (e_E): & (g_G); & (h_H);\end{matrix}\; x, y\right] = \sum_{m,n=0}^{\infty} \frac{[(a_A)]_{m+n}\,[(b_B)]_m\,[(d_D)]_n}{[(e_E)]_{m+n}\,(g_G)_m\,(h_H)_n}\,\frac{x^m}{m!}\,\frac{y^n}{n!}, \tag{1.3}$$

where, for convergence,

(i) $A+B < E+G+1, \quad A+D < E+H+1, \quad |x| < \infty, \quad |y| < \infty.$

or

(ii) $A+B = E+G+1, \quad A+D = E+H+1,$ and

$$\begin{cases} |x|^{1/(A-E)} + |y|^{1/(A-E)} < 1, & if\ A > E, \\ \max\{|x|,\ |y|\} < 1, & if\ A \le E. \end{cases}$$

A general triple hypergeometric series $F^{(3)}$ of Srivastava's [8, p. 69 (39)] defined as

$$F^{(3)}\left[\begin{matrix}(a) :: (b); (b'); (b''); (e); (e');\ (e'') \ ; \\ (d) :: (g); (g'); (g''); (h); (h'); (h'') \ ;\end{matrix}\; x, y, z\right]$$

$$= \sum_{m,n,p=0}^{\infty} \frac{[(a)]_{m+n+p}[(b)]_{m+n}[(b')]_{n+p}[(b'')]_{p+m}[(e)]_m[(e')]_n[(e'')]_p}{[(d)]_{m+n+p}[(g)]_{m+n}[(g')]_{n+p}[(g'')]_{p+m}[(h)]_m[(h')]_n[(h'')]_p}\,\frac{x^m}{m!}\,\frac{y^n}{n!}\,\frac{z^p}{p!}, \tag{1.4}$$

where (a) means $a_1, a_2, ...a_n$ and $((a))_n$ has the interpretation

$$\prod_{j=1}^{A}(a_j)_n = \prod_{j=1}^{A}\frac{\Gamma(a_j+n)}{\Gamma(a_j)}$$

and a hypergeometric function of four variables [6; p. 172 (2)] is given in the form

$$F_P^{(4)}\left[\begin{matrix}a :: b; -; -; -; d; e; f, g; \\ c :: -; -; -; -; d'; e'; f';\end{matrix}\; u, x, y, z\right]$$

$$= \sum_{m,n,p,q=0}^{\infty} \frac{(a)_{m+n+p+q}(b)_{m+n+p}(d)_m(e)_n(f)_p(g)_q}{(c)_{m+n+p+q}(d')_m(e')]_n(f')_p}\,\frac{u^m}{m!}\,\frac{x^m}{n!}\,\frac{y^n}{p!}\,\frac{z^p}{q!}\,. \tag{1.5}$$

## 2. Main Transformation

We establish an integral in the form

$$\int_0^\infty t^{\lambda-\frac{1}{2}}\, e^{-pt}\ {}_1F_2\left(\begin{matrix} b & ; \\ c,d & ; \end{matrix}\ -u^2t^2\right)\, J_{\nu_1}(\beta_1 t)\, J_{\nu_2}(\beta_2 t)\cdots J_{\nu_n}(\beta_n t)\, dt$$

$$= \frac{2^{-(\nu_1+\nu_2...+\nu_n)}\ \beta_1^{\nu_1}......\beta_n^{\nu_n}\ \Gamma(A+2n)}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)...\Gamma(\nu_n+1)\ [p+i(\beta_1+\beta_2...\beta_n)]^{A+2n}} \sum_{n=0}^{\infty} \frac{(b)_n\ (-u^2)^n}{(c)_n\ (d)_n n!}$$

$$F_A^{(n)}\left[A+2n;\nu_1+\frac{1}{2},...,\nu_n+\frac{1}{2};2\nu_1+1,...,2\nu_n+1;\frac{2\beta_1 i}{p+i(\beta_1+\beta_2...\beta_n)},...,\frac{2\beta_n i}{p+i(\beta_1+\beta_2...\beta_n}\right], \tag{2.1}$$

where $i^2=-1,\ A=\lambda+\nu_1+........+\nu_n+\frac{1}{2}, Re[p+i(\beta_1+\beta_2...+\beta_n)]>0, Re(A)>0,$

$F_A^{(n)}$ is called Lauricella function defined by equation (1.1) and $J_\nu(z)$ is the Bessel function ([7] see also [8]) defined as

$$J_\nu(z) = \frac{(z/2)^\nu}{\Gamma(\nu+1)}\ {}_0F_1\left[\begin{matrix} -; & \frac{-z^2}{4} \\ \nu+1; & \end{matrix}\right]. \tag{2.2}$$

Equation (2.1) can be obtained by expanding ${}_1F_2$ in series form and then integrating term by term with the help of integral transform [2; p. 184 (24)].

## 3. Special cases

In this section, we establish some interesting special cases of our main result, which are given as follows:

**1.** On setting $n=1$ in (2.1) and using equation (1.2), we get

$$\int_0^\infty t^{\lambda-\frac{1}{2}}e^{-pt}\ {}_1F_2\left(\begin{matrix} b & ; \\ c,d & ; \end{matrix}\ -u^2t^2\right)\, J_{\nu_1}(\beta_1 t)dt$$

$$= \frac{2^{-(\nu_1)}\ \beta_1^{\nu_1}\ \Gamma(\lambda+\nu_1+2n+\frac{1}{2})}{\Gamma(\nu_1+1)\ [p+i(\beta_1)]^{\lambda+\nu_1+2n+\frac{1}{2}}} \sum_{n=0}^{\infty}\frac{(b)_n\ (-u^2)^n}{(c)_n\ (d)_n n!}$$

$$\times {}_2F_1\left(\begin{matrix} \lambda+\nu_1+2n+\frac{1}{2},\ \nu_1+\frac{1}{2} & ; & \\ & & \frac{2\beta_1 i}{p+i\beta_1} \\ 2\nu_1+1 & ; & \end{matrix}\right), \tag{3.1}$$

where $Re(\lambda+\nu_1+\frac{1}{2})>0,\ Re(p+i\beta_1)>0.$

**2.** On setting $n=2$ in (2.1), we get the known result of Khan and Kashmin [3] as follows:

$$\int_0^\infty t^{\lambda-\frac{1}{2}} e^{-pt}\ {}_1F_2\left(\begin{array}{c} b \ ; \\ c,d \ ; \end{array} -u^2t^2\right) J_{\nu_1}(\beta_1 t)\ J_{\nu_2}(\beta_2 t)dt$$

$$= \frac{2^{-(\nu_1+\nu_2)}\ \beta_1^{\nu_1}\beta_2^{\nu_2}\ \Gamma(\lambda+\nu_1+\nu_2+2n+\frac{1}{2})}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)\ [p+i(\beta_1+\beta_2)]^{\lambda+\nu_1+\nu_2+2n+\frac{1}{2}}} \sum_{n=0}^{\infty} \frac{(b)_n\ (-u^2)^n}{(c)_n\ (d)_n n!}$$

$$\times F_2\left[\lambda+\nu_1+\nu_2+2n+\frac{1}{2};\nu_1+\frac{1}{2},\nu_2+\frac{1}{2};2\nu_1+1,2\nu_2+1;\frac{2\beta_1 i}{p+i(\beta_1+\beta_2)},\frac{2\beta_2 i}{p+i(\beta_1+\beta_2)}\right], \tag{3.2}$$

where $Re(\lambda+\nu_1+\nu_2+\frac{1}{2})>0,\ Re[p+i(\beta_1+\beta_2)]>0$ and $F_2$ is an Appell function [8; p. 53 (5)].

**3.** By using the following known transformation [1; p. 381] ,

$$F_2[\alpha,\beta,\beta',2\beta,2\beta';2x,y] = (1-x)^{-\alpha}\ H_4\left[\alpha,\beta',\beta+\frac{1}{2},2\beta';\frac{x^2}{4(1-x)^2},\frac{y}{1-x}\right], \tag{3.3}$$

equation (3.2) reduces to

$$\int_0^\infty t^{\lambda-\frac{1}{2}} e^{-pt}\ {}_1F_2\left(\begin{array}{c} b \ ; \\ c,d \ ; \end{array} -u^2t^2\right) J_{\nu_1}(\beta_1 t) J_{\nu_2}(\beta_2 t)dt$$

$$= \frac{2^{-(\nu_1+\nu_2)}\beta_1^{\nu_1}\beta_2^{\nu_2}\Gamma(\lambda+\nu_1+\nu_2+2n+\frac{1}{2})}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)[p+i(\beta_1+\beta_2)]^{\lambda+\nu_1+\nu_2+2n+\frac{1}{2}}}$$

$$\times \sum_{n=0}^{\infty} \frac{(b)_n(-u^2)^n}{(c)_n(d)_n n!}\left[1-\frac{\beta_1 i}{p+i(\beta_1+\beta_2)}\right]^{-(\lambda+\nu_1+\nu_2+2n+\frac{1}{2})}$$

$$\times H_4\left[\lambda+\nu_1+\nu_2+2n+\frac{1}{2},\nu_2+\frac{1}{2},\nu_1+1;2\nu_2+1;\left(\frac{\beta_1 i}{4[p+i(\beta_2)]}\right)^2,\frac{2\beta_2 i}{p+i(\beta_2)}\right]. \tag{3.4}$$

**4.** Further by using a known result of Srivastava and Karlsson [9; p. 270 (2)],

$$F_2 = F^{1:1:1}_{0:1:1}\ , \tag{3.5}$$

equation (3.2), becomes

$$\int_0^\infty t^{\lambda-\frac{1}{2}} e^{-pt}\ {}_1F_2\left(\begin{array}{c} b \ ; \\ c,d \ ; \end{array} -u^2t^2\right) J_{\nu_1}(\beta_1 t)\ J_{\nu_2}(\beta_2 t)dt$$

$$= \frac{2^{-(\nu_1+\nu_2)}\ \beta_1^{\nu_1}\beta_2^{\nu_2}\ \Gamma(\lambda+\nu_1+\nu_2+2n+\frac{1}{2})}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)\ [p+i(\beta_1+\beta_2)]^{\lambda+\nu_1+\nu_2+2n+\frac{1}{2}}} \sum_{n=0}^{\infty} \frac{(b)_n\ (-u^2)^n}{(c)_n\ (d)_n n!}$$

$$\times\, F^{1:1:1}_{0:1:1}\left[\begin{array}{l} \lambda+\nu_1+\nu_2+2n+\frac{1}{2};\ \nu_1+\frac{1}{2},\nu_2+\frac{1}{2}\ ; \\ \_\_\_ : 2\nu_1+1, 2\nu_2+1\ ; \end{array} \frac{2\beta_1 i}{p+i(\beta_1+\beta_2)}, \frac{2\beta_2 i}{p+i(\beta_1+\beta_2)}\right], \quad (3.6)$$

where $F^{p:q:s}_{l:m:n}$ is Kamp$e'$ de F$e'$riet function defined by Srivastava and Panda [10].

**5.** On setting $n=3$ in (2.1), we obtain a new result associated with Lauricella's function of three variables $F^{(3)}_A$ as

$$\int_0^\infty t^{\lambda-\frac{1}{2}} e^{-pt}\ {}_1F_2\left(\begin{array}{c} b \\ c,d \end{array} \begin{array}{c} ; \\ ; \end{array} -u^2t^2\right) J_{\nu_1}(\beta_1 t)\ J_{\nu_2}(\beta_2 t)\ J_{\nu_3}(\beta_3 t)\ dt$$

$$= \frac{2^{-(\nu_1+\nu_2+\nu_3)}\ \beta_1^{\nu_1}\beta_2^{\nu_2}\beta_3^{\nu_3}\ \Gamma(\lambda+\nu_1+\nu_2+\nu_3+2n+\frac{1}{2})}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)\Gamma(\nu_3+1)\ [p+i(\beta_1+\beta_2+\beta_3)]^{\lambda+\nu_1+\nu_2+\nu_3+2n+\frac{1}{2}}} \sum_{n=0}^\infty \frac{(b)_n\ (-u^2)^n}{(c)_n\ (d)_n n!}$$

$$\times F^{(3)}_A\left[\lambda+\nu_1+\nu_2+\nu_3+2n+\frac{1}{2}; \nu_1+\frac{1}{2}, \nu_2+\frac{1}{2}, \nu_3+\frac{1}{2}; 2\nu_1+1, 2\nu_2+1, 2\nu_3+1;\right.$$
$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\beta_3)}, \frac{2\beta_2 i}{p+i(\beta_1+\beta_2+\beta_3)}, \frac{2\beta_3 i}{p+i(\beta_1+\beta_2+\beta_3)}\right], \quad (3.7)$$

where $Re(\lambda+\nu_1+\nu_2+\nu_3+\frac{1}{2})>0,\ Re[p+i(\beta_1+\beta_2+\beta_3)]>0$.

In view of a known result of Srivastava and Karlsson [9; p. 271 (11)],

$$F^{(3)}_A[\alpha,\beta_1,\beta_2,\beta_3;\gamma_1,\gamma_2,\gamma_3;x,y,z] = F^{(3)}\left[\begin{array}{l} (\alpha) :: \_;\_;\_;\beta_1,\beta_2,\beta_3; \\ \_ :: \_;\_;\_;\gamma_1,\gamma_2,\gamma_3; \end{array} x,y,z\right], \quad (3.8)$$

equation (3.7) becomes

$$\int_0^\infty t^{\lambda-\frac{1}{2}} e^{-pt}\ {}_1F_2\left(\begin{array}{c} b \\ c,d \end{array} \begin{array}{c} ; \\ ; \end{array} -u^2t^2\right) J_{\nu_1}(\beta_1 t)\ J_{\nu_2}(\beta_2 t)\ J_{\nu_3}(\beta_3 t)\ dt$$

$$= \frac{2^{-(\nu_1+\nu_2+\nu_3)}\ \beta_1^{\nu_1}\beta_2^{\nu_2}\beta_3^{\nu_3}\ \Gamma(\lambda+\nu_1+\nu_2+\nu_3+2n+\frac{1}{2})}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)\Gamma(\nu_3+1)\ [p+i(\beta_1+\beta_2+\beta_3)]^{\lambda+\nu_1+\nu_2+\nu_3+2n+\frac{1}{2}}} \sum_{n=0}^\infty \frac{(b)_n\ (-u^2)^n}{(c)_n\ (d)_n n!}$$

$$\times F^{(3)}\left[\begin{array}{l} \lambda+\nu_1+\nu_2+\nu_3+2n+\frac{1}{2} :: \_;\ \_;\ \_;\ \nu_1+\frac{1}{2},\ \nu_2+\frac{1}{2},\ \nu_3+\frac{1}{2}; \\ \_\_\_\_\_\_ :: \_;\ \_;\ \_;\ 2\nu_1+1,\ 2\nu_2+1,\ 2\nu_2+1; \end{array}\right.$$
$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\beta_3)}, \frac{2\beta_2 i}{p+i(\beta_1+\beta_2+\beta_3)}, \frac{2\beta_3 i}{p+i(\beta_1+\beta_2+\beta_3)}\right], \quad (3.9)$$

where $Re(\lambda+\nu_1+\nu_2+\nu_3+\frac{1}{2})>0,\ Re[p+i(\beta_1+\beta_2+\beta_3)]>0$ and $F^{(3)}$ is triple hypergeometric series defined by equation (1.4).

Further on expanding $F^{(3)}$ into a series form with the help of (1.4), equation (3.9) reduces to

$$\int_0^\infty t^{\lambda-\frac{1}{2}}e^{-pt}\ {}_1F_2\left(\begin{matrix} b & ; \\ c,d & ; \end{matrix}\ -u^2t^2\right) J_{\nu_1}(\beta_1 t)\, J_{\nu_2}(\beta_2 t)\, J_{\nu_3}(\beta_3 t)\, dt$$

$$=\frac{2^{-(\nu_1+\nu_2+\nu_3)}\ \beta_1^{\nu_1}\beta_2^{\nu_2}\beta_3^{\nu_3}\ \Gamma(\lambda+\nu_1+\nu_2+\nu_3+\frac{1}{2})}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)\Gamma(\nu_3+1)\ [p+i(\beta_1+\beta_2+\beta_3)]^{\lambda+\nu_1+\nu_2+\nu_3+\frac{1}{2}}}$$

$$\times F_p^{(4)}\left[\begin{matrix} \lambda+\nu_1+\nu_2+\nu_3+\frac{1}{2} :: & \_; & \_; & \_; & \nu_1+\frac{1}{2}, & \nu_2+\frac{1}{2}, & \nu_3+\frac{1}{2}; & b; \\ \_\_\_\_ \quad :: & \_; & \_; & \_; & 2\nu_1+1, & 2\nu_2+1, & 2\nu_3+1; & c,\ d; \end{matrix}\right.$$

$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\beta_3)},\frac{2\beta_2 i}{p+i(\beta_1+\beta_2+\beta_3)},\frac{2\beta_3 i}{p+i(\beta_1+\beta_2+\beta_3)},\frac{-u^2}{p+i(\beta_1+\beta_2+\beta_3)}\right], \tag{3.10}$$

where $Re(\lambda+\nu_1+\nu_2+\nu_3+\frac{1}{2})>0,\ Re[p+i(\beta_1+\beta_2+\beta_3)]>0$ and $F_P^{(4)}$ is the hypergeometric function of four variables defined by Pathan [6; p. 1721.2].

**6.** On setting $c=b+\frac{1}{2},\ d=2b-1$ in (2.1) and using a known result [5; p. 595 (6)], we get

$$\Gamma\left(b-\frac{1}{2}\right)\Gamma\left(b+\frac{1}{2}\right)\left(\frac{-u^2}{2}\right)^{2-2b}\int_0^\infty t^{\lambda-4b+\frac{7}{2}}e^{-pt}[I_{b-\frac{3}{2}}(-u^2t^2)I_{b-\frac{1}{2}}(-u^2t^2)]J_{\nu_1}(\beta_1 t)$$

$$\times J_{\nu_2}(\beta_2 t)\cdots J_{\nu_n}(\beta_n t)\,dt=\frac{2^{-(\nu_1+\nu_2...+\nu_n)}\ \beta_1^{\nu_1}......\beta_n^{\nu_n}\ \Gamma(A+2n)}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)...\Gamma(\nu_n+1)\ [p+i(\beta_1+\beta_2+\cdots+\beta_n)]^{A+2n}}$$

$$\times\sum_{n=0}^\infty\frac{(b)_n\ (-u^2)^n}{(b+\frac{1}{2})_n\ (2b-1)_n n!}\ F_A^{(n)}\left[A+2n;\nu_1+\frac{1}{2},...,\nu_n+\frac{1}{2};2\nu_1+1,...,2\nu_n+1;\right.$$

$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)},\cdots,\frac{2\beta_n i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)}\right], \tag{3.11}$$

where $A=\lambda+\nu_1+........+\nu_n+\frac{1}{2}, Re[p+i(\beta_1+\beta_2...+\beta_n)]>0, Re(A)>0$ and $I_\nu$ is the modified Bessel function [8].

**7.** On setting $b=\frac{3}{2}, c=b,\ d=3-b$ in (2.1) and using the known result [5; p. 595 (10)], we get

$$\frac{\pi(c-1)(c-2)}{\sin c\pi}\int_0^\infty t^{\lambda-\frac{1}{2}}e^{-pt}[I_{1-c}(-u^2t^2)I_{c-1}(-u^2t^2)+I_{2-c}(-u^2t^2)I_{c-2}(-u^2t^2)]J_{\nu_1}(\beta_1 t)$$

$$\times J_{\nu_2}(\beta_2 t)\cdots J_{\nu_n}(\beta_n t)dt=\frac{2^{-(\nu_1+\nu_2...+\nu_n)}\ \beta_1^{\nu_1}......\beta_n^{\nu_n}\ \Gamma(A+2n)}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)...\Gamma(\nu_n+1)\ [p+i(\beta_1+\beta_2+\cdots+\beta_n)]^{A+2n}}$$

$$\times\sum_{n=0}^{\infty}\frac{(\frac{3}{2})_n\ (-u^2)^n}{(b)_n\ (3-b)_n n!}\ F_A^{(n)}\left[A+2n;\nu_1+\frac{1}{2},...,\nu_n+\frac{1}{2};2\nu_1+1,...,2\nu_n+1;\right.$$

$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)},...,\frac{2\beta_n i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)}\right],\qquad(3.12)$$

where $A=\lambda+\nu_1+........+\nu_n+\frac{1}{2}, Re[p+i(\beta_1+\beta_2...+\beta_n)]>0, Re(A)>0.$

**8.** On setting $b=\frac{1}{4},\ c=\frac{1}{2},\ d=\frac{5}{4}$ in (2.1) and using the known result [5; p. 598 (37)], we get

$$\frac{1}{4}\sqrt{\frac{\pi}{-2u^2}}\int_0^\infty t^{\lambda-3/2}e^{-pt}\ [\mathrm{erf}(\sqrt{-2u^2t^2})+\mathrm{erfi}(\sqrt{-2u^2t^2})]J_{\nu_1}(\beta_1 t)J_{\nu_2}(\beta_2 t)\cdots J_{\nu_n}(\beta_n t)\ dt$$

$$=\frac{2^{-(\nu_1+\nu_2...+\nu_n)}\ \beta_1^{\nu_1}......\beta_n^{\nu_n}\ \Gamma(A+2n)}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)...\Gamma(\nu_n+1)\ [p+i(\beta_1+\beta_2+\cdots+\beta_n)]^{A+2n}}\sum_{n=0}^{\infty}\frac{(\frac{1}{4})_n\ (-u^2)^n}{(\frac{1}{2})_n\ (\frac{5}{4})_n n!}$$

$$\times F_A^{(n)}\left[A+2n;\nu_1+\frac{1}{2},...,\nu_n+\frac{1}{2};2\nu_1+1,...,2\nu_n+1;\right.$$

$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)},...,\frac{2\beta_n i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)}\right],\qquad(3.13)$$

where $A=\lambda+\nu_1+........+\nu_n+\frac{1}{2}, Re[p+i(\beta_1+\beta_2...+\beta_n)]>0, Re(A)>0$ and erf is the Error function [8].

**9.** On setting $b=1, c=\frac{3}{2}$, and $d=b$ in (2.1) and using the relations [5; p. 595 (11)] and integral transform [2; p. 184 (24)], we get

$$\frac{\sqrt{\pi}}{2}\ \Gamma(c)(-u^2)^{1/2-c}\int_0^\infty t^{\lambda-2c+1/2}\ e^{-pt}\ [L_{c-3/2}(-2u^2t^2)]J_{\nu_1}(\beta_1 t)J_{\nu_2}(\beta_2 t)\cdots J_{\nu_n}(\beta_n t)\ dt$$

$$=\frac{2^{-(\nu_1+\nu_2...+\nu_n)}\ \beta_1^{\nu_1}......\beta_n^{\nu_n}\ \Gamma(A+2n)}{\Gamma(\nu_1+1)\Gamma(\nu_2+1)...\Gamma(\nu_n+1)\ [p+i(\beta_1+\beta_2+\cdots+\beta_n)]^{A+2n}}\sum_{n=0}^{\infty}\frac{(-u^2)^n}{(\frac{3}{2})_n\ n!}$$

$$\times F_A^{(n)}\left[A+2n;\nu_1+\frac{1}{2},...,\nu_n+\frac{1}{2};2\nu_1+1,...,2\nu_n+1;\right.$$

$$\left.\frac{2\beta_1 i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)},...,\frac{2\beta_n i}{p+i(\beta_1+\beta_2+\cdots+\beta_n)}\right],\qquad(3.14)$$

where $A = \lambda + \nu_1 + ........ + \nu_n + \frac{1}{2}, Re[p + i(\beta_1 + \beta_2 ... + \beta_n)] > 0, Re(A) > 0$ and $L_\nu$ is the Struve function [8].

## 4. Series Expansion

The generalized hypergeometric series of power t is given by [5; p. 418 (10)],

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \, {}_{p+2}F_q \left( \begin{matrix} \frac{(-n)}{2}, \frac{1-n}{2}, (a_p) \; ; \\ (b_q) \; ; \end{matrix} \; u \right) = e^t \; {}_pF_q \left( \begin{matrix} (a_p) \; ; \\ (b_q) \; ; \end{matrix} \; \frac{ut^2}{4} \right) \tag{4.1}$$

For p=1, q=2 equation (4.1) reduces to

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} \, {}_3F_2 \left( \begin{matrix} \frac{(-n)}{2}, \frac{1-n}{2}, (a_1) \; ; \\ (b_1), (b_2) \; ; \end{matrix} \; u \right) = e^t \; {}_1F_2 \left( \begin{matrix} (a_1) \; ; \\ (b_1), (b_2) \; ; \end{matrix} \; \frac{ut^2}{4} \right) \tag{4.2}$$

Multiplying both side of (4.2) by $t^{\lambda - \frac{1}{2}} e^{-pt} \; J_{\nu_1}(\beta_1 t) J_{\nu_2}(\beta_2 t) \cdots J_{\nu_n}(\beta_n t)$ and integrating term by term with the help of result [2; p. 184 (24)], we obtain following generating relation

$$\sum_{n=0}^{\infty} \frac{(A)_n}{p^n n!} \; F_A^{(n+1)} \left[ \frac{A+n}{2}, \frac{A+n}{2} + \frac{1}{2}; -\frac{n}{2}, \frac{1-n}{2}, a_1; \nu_1 + 1, ..., \nu_n + 1; b_1, b_2; \right.$$
$$\left. \frac{-{\beta_1}^2}{p^2}, \frac{-{\beta_2}^2}{p^2} ..., \frac{-{\beta_n}^2}{p^2}, u \right]$$
$$= \left( \frac{p}{p-1} \right)^A \; F_A^{(n+1)} \left[ \frac{A}{2}, \frac{A+1}{2}; a_1; \nu_1 + 1, ..., \nu_n + 1; b_1, b_2; \right.$$
$$\left. \frac{-{\beta_1}^2}{(p-1)^2}, \frac{-{\beta_2}^2}{(p-1)^2} ..., \frac{-{\beta_n}^2}{(p-1)^2}, \frac{u}{4(p-1)^2} \right]. \tag{4.3}$$

If we set $u = 0$ in (4.3), we get

$$\sum_{n=0}^{\infty} \frac{(A)_n}{p^n n!} \; F_A^{(n)} \left[ \frac{A+n}{2}, \frac{A+n}{2} + \frac{1}{2}; \nu_1 + 1, ..., \nu_n + 1; \frac{-{\beta_1}^2}{p^2}, \frac{-{\beta_2}^2}{p^2}, \cdots, \frac{-{\beta_n}^2}{p^2} \right]$$
$$\left( \frac{p}{p-1} \right)^A \; F_A^{(n)} \left[ \frac{A}{2}, \frac{A+1}{2}; \nu_1 + 1, ..., \nu_n + 1; \frac{-{\beta_1}^2}{(p-1)^2}, \frac{-{\beta_2}^2}{(p-1)^2}, \cdots, \frac{-{\beta_n}^2}{(p-1)^2} \right]. \tag{4.4}$$

## Concluding Remark

In the present investigation, we have established an integral transformation involving Bessel function $J_\nu(z)$ of first kind into a multiple hypergeometric series of Lauricella function $F_A^{(n)}$ of $n$ variables, which generalizes a number of known and new transformation for a hypergeometric function ${}_2F_1$, Appell function $F_2$, Lauricella function $F_A^{(3)}$ and the hypergeometric function of four variables $F_P^{(4)}$. The results appear in the paper may be found useful in some areas of Mathematical Physics and Engineering.

## References

[1] Erdelyi, A. et al : *Transformation of hypergeometric functions of two variables,* Proc. Roy, Soc. Edinburgh, Sect.A. **62** (1948), 378-385.

[2] Erdelyi, A. et al : *Table of integral transforms,* Vol.I, McGraw-Hill, New York ,(1954).

[3] Khan, N. U. and Kashmin. T. : Integral Transforms involving Bessel's Function, Communicated to Honam Mathematical Journal.

[4] Lauricella, G. : *Sulle funzioni ipergeometriche a piu variabili* , Rend. Circ. Mat. Palermo, **7**, 111-158 (1893).

[5] Prudnikov. A. P. et al ; *Integrals and Series,* Vol.3, More special functions, Gorden and Breach Science Publishers, New York,(1990).

[6] Pathan, M. A. : *On a transformation of a general hypergeometric series of four variables,* Nederl , Akad. Wetensch. Proc Ser . A. 82=Indag Math 41, New York ,(1979), 171-175.

[7] Rainville. E. D. : *Special functions,* The MacMillan Comp., New York, (1960).

[8] Srivastava. H. M. and Manocha. H. L. : *A Treatise on generating functions,* Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and sons, New York,(1984).

[9] Srivastav. H. M. and Karlsson. P. W.: *Multiple Gaussian hypergeometric series,* Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and sons, New York, Chichester Brisbane, Toronto (1985).

[10] Srivastav. H. M. and Panda, R. : *An integral representation for the product of two Jacobi polynomials,* J. London Math. Soc (2) 12, (1976), 419-425.

Department of Applied Mathematics,
Faculty of Engineering and Technology
Aligarh Muslim University
Aligarh - 202002, U.P. India.
*E-mail address*: nukhan.math@gmail.com
*E-mail address*: talhausman.maths@gmail.com
*E-mail address*: ghayas.maths@gmail.com