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$\varphi-$ CONFORMALLY FLAT $(LCS)_n$ - MANIFOLDS

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Abstract The object of the present paper is to study $\varphi-$ conformally flat, $\varphi-$ conharmonically flat, $\varphi-$ projectively flat and $\varphi-$ concircularly flat $(LCS)_n-$ manifolds. In each of these cases, we obtain that the manifold is an $\eta-$ Einstein manifold.

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1. Introduction

In 2003, A. A. Shaikh ([1]) introduced the notion of Lorentzian concircular structure manifolds (briefly $(LCS)_n-$ manifolds) with an example. An n-dimensional Lorentzian manifold M is a smooth connected para-compact Housdorff manifold with a Lorentzian metric g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_p(M) \times T_p(M) \to R$ is a non-degenerate inner product of signature $(-,+,+,.......,+)$, where $T_p(M)$ denotes the tangent vector space of M at p and R is the real number space. A non-zero vector $v \in T_p(M)$ is said to be time like (resp. non-space like, null like, space like) if it satisfies $g_p(v,v) < 0$ (resp. $\leq 0, = 0, > 0$) ([1], [4]).

Let $(M^n, g),\ n = dimM^n > 3$, be a connected semi Riemannian manifold of class C^∞ and ∇ be its Levi-Civita connection. The Riemannian-Christoffel curvature tensor R, the Weyl conformal curvature tensor C , the conharmonic curvature tensor L , the projective curvature tensor P and the concircular curvature tensor V of (M^n, g) are defined by ([12], [15])

(1.1) $$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z,$$

(1.2) $$C(X,Y)Z = R(X,Y)Z + \frac{1}{n-2}[S(X,Z)Y - S(Y,Z)X + g(X,Z)QY - g(Y,Z)QX] - \frac{r}{(n-1)(n-2)}[g(X,Z)Y - g(Y,Z)X],$$

(1.3) $$L(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY],$$

(1.4) $$P(X,Y)Z = R(X,Y)Z - \tfrac{1}{n-1}[g(Y,Z)QX - g(X,Z)QY],$$

(1.5) $$V(X,Y)Z = R(X,Y)Z - \tfrac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y],$$

respectively, where Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$, S is the Ricci tensor, $r = tr(S)$ is the scalar curvature and $X, Y, Z \in \chi(M)$, $\chi(M)$ is being Lie algebra of vector fields of M.

Recently Arslan, Murathan and Özgür ([14]) studied $\varphi-$ conformally flat (κ, μ) contact metric manifolds, Özgür ([5]) studied $\varphi-$ conformally flat Lorentzian para-Sasakian manifolds and Yildiz, Turan and Murathan ([2]) studied on a Lorentzian $\alpha-$ Sasakian manifolds. $(LCS)_n$ - manifold is studied by D. G. Prakasha ([6], [8]), A. A. Shaikh, T. Basu and S. Eyasmin ([3]) and so many geometers ([7], [9], [16]).

The paper is organized as follows : Section 1 is introductory, In section 2, we give the basic definition of Lorentzian concircular structure manifolds with an example and in section 3, 4, 5 and 6, we study $\varphi-$ conformally flat $(LCS)_n-$ manifold, $\varphi-$ conharmonically flat $(LCS)_n-$ manifold, $\varphi-$ projectively flat $(LCS)_n-$ manifold and $\varphi-$ concircularly flat $(LCS)_n-$ manifold respectively and proved that in each of these cases, the manifold is an $\eta-$ Einstein manifold.

2. Preliminaries

Let M^n be a Lorentzian manifold admitting a unit time like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

(2.1) $$g(\xi, \xi) = -1.$$

Since ξ is a unit concircular vector field, there exists a non-zero 1-form η such that for

(2.2) $$g(X, \xi) = \eta(X),$$

the equation of the following form holds

(2.3) $$(\nabla_X \eta)(Y) = \alpha[g(X, Y) + \eta(X)\eta(Y)], \alpha \neq 0,$$

for all vector fields X, Y where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfies

(2.4) $$(\nabla_X \alpha) = (X\alpha) = \alpha(X) = \rho\, \eta(X),$$

where ρ being a certain scalar function. By virtu of (2.2), (2.3) and (2.4) it follows that

(2.5) $$(X\rho) = d\rho(X) = \beta\, \eta(X),$$

where $\beta = -(\xi, \rho)$ is a scalar function.

Next if we put

(2.6) $$\varphi X = \frac{1}{\alpha} \nabla_X \xi.$$

Then from (2.3) and (2.6), we have

(2.7) $$\varphi X = X + \eta(X)\xi,$$

from which it follows that φ is a symmetric (1, 1) tensor field and is called structure tensor of the manifold. Thus the Lorentzian manifold M together with the unit time like concircular vector field ξ, its associated 1-form η and (1, 1) tensor field φ is said to be a Lorentzian concircular structure manifolds (briefly $(LCS)_n-$ manifolds) ([1]). Especially, if we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto ([13]).

In an $(LCS)_n-$ manifold, the following relations hold ([1], [8]):

(2.8)(a) $\eta(\xi) = -1$ (b) $\varphi\xi = 0$ (c) $\eta(\varphi X) = 0$,

(2.9) $$g(\varphi X, \varphi Y) = g(X, Y) + \eta(X)\eta(Y),$$

(2.10) $$(\nabla_X \varphi)(Y) = \alpha[g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi],$$

(2.11) $$g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = (\alpha^2 - \rho)[g(Y, Z)\eta(X) - g(X, Z)\eta(Y)],$$

(2.12) $R(X, Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y],$
(2.13) $R(\xi, X)Y = (\alpha^2 - \rho)[g(X, Y)\xi - \eta(Y)X],$
(2.14) $R(\xi, X)\xi = (\alpha^2 - \rho)[X + \eta(X)\xi],$
(2.15) $S(X, \xi) = (n-1)(\alpha^2 - \rho)\eta(X),$
(2.16) $Q\xi = (n-1)(\alpha^2 - \rho)\xi,$
and
(2.17) $S(\varphi X, \varphi Y) = S(X, Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y),$
for all vector fields X, Y, Z, where R, S denote respectively the curvature tensor and the Ricci tensor of the manifold. The example of a Lorentzian concircular structure manifods is given by Shaikh ([1]):

Example 2.1. Let R^4 be the 4-dimensional real number space with coordinates (x_1, x_2, x_3, x_4). If we put ([1])

$$\varphi = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$g = \frac{1}{4}\begin{bmatrix} 1+x_2^2 & 0 & 0 & 0 \\ 0 & 1+x_3^2 & 0 & 0 \\ 0 & 0 & 1+x_2^2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

$$\xi = (\xi^h) = \begin{bmatrix} 0 & 0 & 0 & -2 \end{bmatrix}^t,$$

$$\eta = (\eta_i) = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} \end{bmatrix},$$

then it can be easily seen that the quadruple (φ, ξ, η, g) defines a four dimensional Lorentzian concircular structure manifods.

Definition 2.1. An $(LCS)_n-$ manifold M^n is said to be an $\eta-$Einstein manifold if its Ricci tensor S is of the form

$$S(X, Y) = \alpha g(X, Y) + \beta\eta(X)\,\eta(Y),$$

for any vector fields X and Y, where α, β are smooth functions on M^n.

3. $\varphi-$ conformally flat $(LCS)_n-$ manifold.

In this section we consider $\varphi-$ conformally flat $(LCS)_n-$ manifold.

Let C be the Weyl conformal curvature tensor of M^n. Since at each point $p \in M^n$ the tangent space $T_p(M^n)$ can be decomposed into direct sum $T_p(M^n) = \varphi(T_p(M^n)) \oplus L(\xi_p)$, where $L(\xi_p)$ is a $1-$ dimensional linear subspace of $T_p(M^n)$ generated by ξ_p, we have map:

$C : T_p(M^n) \times T_p(M^n) \times T_p(M^n) \to \varphi(T_p(M^n)) \oplus L(\xi_p).$

It may natural to consider the following particular cases:

(1) $C : T_p(M^n) \times T_p(M^n) \times T_p(M^n) \to L(\xi_p)$, that is, the projection of the image of C in $\varphi(T_p(M^n))$ is zero.

(2) $C : T_p(M^n) \times T_p(M^n) \times T_p(M^n) \to \varphi(T_p(M^n))$, that is, the projection of the image of C in $L(\xi_p)$ is zero.

(3) $C : \varphi(T_p(M^n)) \times \varphi(T_p(M^n)) \times \varphi(T_p(M^n)) \to L(\xi_p)$, that is, when C is restricted to $\varphi(T_p(M^n)) \times \varphi(T_p(M^n)) \times \varphi(T_p(M^n))$, the projection of the image of C in $\varphi(T_p(M^n))$ is zero. This condition is equivalent to ([11])

(3.1) $$\varphi^2 C(\varphi X, \varphi Y)\varphi Z = 0,$$

Definition 3.1. A differentiable manifold (M^n, g), $n > 3$, satisfying the condition (3.1) is called $\varphi-$ conformally flat $(LCS)_n-$ manifold.

Suppose that $(M^n, g)(n > 3)$, is a $\varphi-$ conformally flat $(LCS)_n-$ manifold. It is easy to see that $\varphi^2 C(\varphi X, \varphi Y)\varphi Z = 0$, holds if and only if

(3.2) $$g(C(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0,$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$.

So by the use of (1.2), $\varphi-$ conformally flat means

(3.3) $$\begin{aligned} g(R(\varphi X, \varphi Y)\varphi Z, \varphi W) = &\frac{1}{(n-2)}[S(\varphi X, \varphi W)g(\varphi Y, \varphi Z) - S(\varphi Y, \varphi W)g(\varphi X, \varphi Z) \\ &+ S(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - S(\varphi X, \varphi Z)g(\varphi Y, \varphi W)] \\ &- \frac{r}{(n-1)(n-2)}[g(\varphi Y, \varphi Z)g(\varphi X, \varphi W) - g(\varphi X, \varphi Z)g(\varphi Y, \varphi W)]. \end{aligned}$$

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1, \varphi e_2, ..., \varphi e_{n-1}, \xi\}$ is also a local orthonormal basis, if we put $X = W = e_i$ in (3.3) and sum up with respect to i, then we have

(3.4)
$$\begin{aligned} \sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = &\frac{1}{(n-2)} \sum_{i=1}^{n-1} [S(\varphi e_i, \varphi e_i)g(\varphi Y, \varphi Z) - S(\varphi Y, \varphi e_i)g(\varphi e_i, \varphi Z) \\ &+ S(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - S(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)] \\ &- \frac{r}{(n-1)(n-2)} \sum_{i=1}^{n-1} [g(\varphi Y, \varphi Z)g(\varphi e_i, \varphi e_i) - g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i)]. \end{aligned}$$

It can be easily verify by straight forward calculation that

(3.5) $$\sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = S(\varphi Y, \varphi Z) + g(\varphi Y, \varphi Z),$$

(3.6) $$\sum_{i=1}^{n-1} S(\varphi e_i, \varphi e_i) = r - (n-1)(\alpha^2 - \rho),$$

(3.7) $$\sum_{i=1}^{n-1} g(\varphi e_i, \varphi Z)S(\varphi Y, \varphi e_i) = S(\varphi Y, \varphi Z),$$

(3.8) $$\sum_{i=1}^{n-1} g(\varphi e_i, \varphi e_i) = n-1,$$

(3.9) $$\sum_{i=1}^{n-1} g(\varphi e_i, \varphi Z)g(\varphi Y, \varphi e_i) = g(\varphi Y, \varphi Z).$$

So by virtue of (3.5)-(3.9), the equation (3.4) takes the form

(3.10) $$S(\varphi Y, \varphi Z) = \Big[\frac{r}{(n-1)} - (n-1)(\alpha^2 - \rho) - (n-2)\Big] g(\varphi Y, \varphi Z).$$

By making the use of (2.9) and (2.17), the equation (3.10) takes the form

$$(3.11)\qquad S(Y,Z)=\Big[\frac{r}{(n-1)}-(n-1)(\alpha^2-\rho)-(n-2)\Big]g(Y,Z)$$
$$+\Big[\frac{r}{(n-1)}-2(n-1)(\alpha^2-\rho)-(n-2)\Big]\eta(Y)\eta(Z).$$

Which shows that M^n is an $\eta-$Einstein manifold.
Hence, we can state the following theorem :

Theorem 3.1. *Let (M^n,g) be an n-dimensional, $(n>3)$, $\varphi-$conformally flat $(LCS)_n-$ manifold, then M^n is an $\eta-$Einstein manifold.*

4. $\varphi-$ conharmonically flat $(LCS)_n-$ manifold

In this section we consider $\varphi-$ conharmonically flat $(LCS)_n-$ manifold.

Definition 4.1. *A differentiable manifold (M^n,g), $n>3$, satisfying the condition $\varphi^2L(\varphi X,\varphi Y)\varphi Z=0$, is called $\varphi-$ conharmonically flat $(LCS)_n-$ manifold.*

Suppose that $(M^n,\,g)(n>3)$, is a $\varphi-$ conharmonically flat $(LCS)_n-$ manifold. It is easy to see that $\varphi^2L(\varphi X,\varphi Y)\varphi Z=0$, holds if and only if

$$(4.1)\qquad g(L(\varphi X,\varphi Y)\varphi Z,\varphi W)=0,$$

for any vector fields $X,Y,Z,W\in\chi(M^n)$.

So by the use of (1.3), $\varphi-$ conharmonically flat means

$$(4.2)\quad g(R(\varphi X,\varphi Y)\varphi Z,\varphi W)=\frac{1}{(n-2)}[S(\varphi X,\varphi W)g(\varphi Y,\varphi Z)-S(\varphi Y,\varphi W)g(\varphi X,\varphi Z)$$
$$+S(\varphi Y,\varphi Z)g(\varphi X,\varphi W)-S(\varphi X,\varphi Z)g(\varphi Y,\varphi W)].$$

Let $\{e_1,e_2,......,e_{n-1},\xi\}$ be a local orthonormal basis of vector fields in M^n. By using the fact that $\{\varphi e_1,\varphi e_2,....,\varphi e_{n-1},\xi\}$ is also a local orthonormal basis, if we put $X=W=e_i$ in (4.2) and sum up with respect to i, then we have

(4.3)
$$\sum_{i=1}^{n-1}g(R(\varphi e_i,\varphi Y)\varphi Z,\varphi e_i)=\frac{1}{(n-2)}\sum_{i=1}^{n-1}[S(\varphi e_i,\varphi e_i)g(\varphi Y,\varphi Z)-S(\varphi Y,\varphi e_i)g(\varphi e_i,\varphi Z)$$
$$+S(\varphi Y,\varphi Z)g(\varphi e_i,\varphi e_i)-S(\varphi e_i,\varphi Z)g(\varphi Y,\varphi e_i)].$$

So by the use of (3.5)-(3.8), the equation (4.3) takes the form

$$(4.4)\qquad S(\varphi Y,\varphi Z)=\Big[r-(n-1)(\alpha^2-\rho)-(n-2)\Big]g(\varphi Y,\varphi Z).$$

By making the use of (2.9) and (2.17), the equation (4.4) takes the form

$$(4.5)\qquad S(Y,Z)=\Big[r-(n-1)(\alpha^2-\rho)-(n-2)\Big]g(Y,Z)$$
$$+\Big[r-2(n-1)(\alpha^2-\rho)-(n-2)\Big]\eta(Y)\eta(Z).$$

Which shows that M^n is an $\eta-$Einstein manifold.
Again putting $Y=Z=e_i$ in (4.5) and sum up with respect to $i, 1\le i\le n$, we get

$$r=(n-1)[(\alpha^2-\rho)+1].$$

Hence, we can state the following theorem :
Theorem 4.1. *Let* (M^n, g) *be an n-dimensional, (n > 3),* $\varphi-$*conharmonically flat* $(LCS)_n-$ *manifold, then* M^n *is an* $\eta-$*Einstein manifold with scalar curvature* $r = (n-1)[(\alpha^2 - \rho) + 1]$.

5. $\varphi-$ *projectively flat* $(LCS)_n-$ *manifold*

In this section we consider $\varphi-$ *projectively flat* $(LCS)_n-$ *manifold.*
Definition 5.1. *A differentiable manifold* (M^n, g), $n > 3$, *satisfying the condition* $\varphi^2 P(\varphi X, \varphi Y)\varphi Z = 0$, *is called* $\varphi-$ *projectively flat* $(LCS)_n-$ *manifold.*

Suppose that $(M^n, g)(n > 3)$, *is a* $\varphi-$ *projectively flat* $(LCS)_n-$ *manifold. It is easy to see that* $\varphi^2 P(\varphi X, \varphi Y)\varphi Z = 0$, *holds if and only if*

$$g(P(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0, \tag{5.1}$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$.
So by the use of (1.4), $\varphi-$ *projectively flat means*

$$g(R(\varphi X, \varphi Y)\varphi Z, \varphi W) = \frac{1}{(n-1)}[S(\varphi X, \varphi W)g(\varphi Y, \varphi Z) - S(\varphi Y, \varphi W)g(\varphi X, \varphi Z)]. \tag{5.2}$$

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ *be a local orthonormal basis of vector fields in* M^n. *By using the fact that* $\{\varphi e_1, \varphi e_2, ..., \varphi e_{n-1}, \xi\}$ *is also a local orthonormal basis, if we put* $X = W = e_i$ *in (5.2) and sum up with respect to i, then we have*

$$\sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = \frac{1}{(n-1)} \sum_{i=1}^{n-1} [S(\varphi e_i, \varphi e_i)g(\varphi Y, \varphi Z) - S(\varphi Y, \varphi e_i)g(\varphi e_i, \varphi Z)]. \tag{5.3}$$

So by the use of (3.5)-(3.7), the equation (5.3) takes the form

$$S(\varphi Y, \varphi Z) = \Big[\frac{r - (n-1)(\alpha^2 - \rho) - (n-1)}{n}\Big] g(\varphi Y, \varphi Z). \tag{5.4}$$

By making the use of (2.9) and (2.17), the equation (5.4) takes the form

$$S(Y, Z) = \Big[\frac{r - (n-1)(\alpha^2 - \rho) - (n-1)}{n}\Big] g(Y, Z) + \Big[\frac{r - (n-1)(\alpha^2 - \rho) - (n-1)}{n} - (n-1)(\alpha^2 - \rho)\Big]\eta(Y)\eta(Z). \tag{5.5}$$

Which shows that M^n *is an* $\eta-$*Einstein manifold.*
Again putting $Y = Z = e_i$ *in (5.5) and sum up with respect to* $i, 1 \le i \le n$, *we get*

$$r = (n-1)[(\alpha^2 - \rho) - (n-1)].$$

Hence, we can state the following theorem :
Theorem 5.1. *Let* (M^n, g) *be an n-dimensional, (n > 3),* $\varphi-$*projectively flat* $(LCS)_n-$ *manifold, then* M^n *is an* $\eta-$*Einstein manifold with scalar curvature* $r = (n-1)[(\alpha^2 - \rho) - (n-1)]$.

6. $\varphi-$ *concircularly flat* $(LCS)_n-$ *manifold*

In this section we consider $\varphi-$ *concircularly flat* $(LCS)_n-$ *manifold.*
Definition 6.1. *A differentiable manifold* (M^n, g), $n > 3$, *satisfying the condition* $\varphi^2 V(\varphi X, \varphi Y)\varphi Z = 0$, *is called* $\varphi-$ *concircularly flat* $(LCS)_n-$ *manifold.*

Suppose that $(M^n, g)(n > 3)$, *is a* $\varphi-$ *concircularly flat* $(LCS)_n-$ *manifold. It is easy to* *see that* $\varphi^2 V(\varphi X, \varphi Y)\varphi Z = 0$, *holds if and only if*

$$g(V(\varphi X, \varphi Y)\varphi Z, \varphi W) = 0, \tag{6.1}$$

for any vector fields $X, Y, Z, W \in \chi(M^n)$.

So by the use of (1.5), $\varphi-$ *concircularly flat means*

$$g(R(\varphi X, \varphi Y)\varphi Z, \varphi W) = \frac{r}{n(n-1)}[g(\varphi X, \varphi W)g(\varphi Y, \varphi Z) - g(\varphi Y, \varphi W)g(\varphi X, \varphi Z)]. \tag{6.2}$$

Let $\{e_1, e_2, ..., e_{n-1}, \xi\}$ *be a local orthonormal basis of vector fields in* M^n. *By using the fact that* $\{\varphi e_1, \varphi e_2, ..., \varphi e_{n-1}, \xi\}$ *is also a local orthonormal basis, if we put* $X = W = e_i$ *in (5.2) and sum up with respect to i, then we have*

$$\sum_{i=1}^{n-1} g(R(\varphi e_i, \varphi Y)\varphi Z, \varphi e_i) = \frac{r}{n(n-1)} \sum_{i=1}^{n-1} [g(\varphi e_i, \varphi e_i)g(\varphi Y, \varphi Z) - g(\varphi Y, \varphi e_i)g(\varphi e_i, \varphi Z)]. \tag{6.3}$$

So by the use of (3.5), (3.8) and (3.9), the equation (6.3) takes the form

$$S(\varphi Y, \varphi Z) = \Big[\frac{r(n-2)}{n(n-1)} - 1\Big]g(\varphi Y, \varphi Z). \tag{6.4}$$

By making the use of (2.9) and (2.17), the equation (6.4) takes the form

$$S(Y, Z) = \Big[\frac{r(n-2)}{n(n-1)} - 1\Big]g(Y, Z) + \Big[\frac{r(n-2)}{n(n-1)} - (n-1)(\alpha^2 - \rho) - 1\Big]\eta(Y)\eta(Z). \tag{6.5}$$

Which shows that M^n *is an* $\eta-$*Einstein manifold.*
Hence, we can state the following theorem :

Theorem 6.1. *Let* (M^n, g) *be an n-dimensional,* $(n > 3)$, $\varphi-$*concircularly flat* $(LCS)_n-$ *manifold, then* M^n *is an* $\eta-$*Einstein manifold.*

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