Global Journal of Advanced Research
on Classical and Modern Geometries
ISSN: 2284-5569, Vol.4, (2015), Issue 2, pp.116-124

SCREEN SEMI-INVARIANT HALF-LIGHTLIKE SUBMANIFOLDS OF A SEMI-RIEMANNIAN PRODUCT MANIFOLD

OĞUZHAN BAHADIR

ABSTRACT. In this paper, we study half-lightlike submanifolds of a semi-Riemannian product manifold. We introduce a classes half-lightlike submanifolds of called screen semi-invariant half-lightlike submanifolds. We defined some special distribution of screen semi-invariant half-lightlike submanifold. We give some equivalent conditions for integrability of distributions with respect to the Levi-Civita connection of semi-Riemannian manifolds and some results.

1. INTRODUCTION

The theory of degenerate submanifolds of semi-Riemannian manifolds is one of a important topics of diferential geometry. The geometry of lightlike submanifolds a semi-Riemannian manifold was presented in [1] (see also [2]) by K.L. Duggal and A. Bejancu. Differential Geometry of Lightlike Submanifolds was presented in [11] by K. L. Duggal and B. Sahin. In [6],[7], [8], [9], K. L. Duggal and B. Sahin introduced and studied geometry of classes of lightlike submanifolds in indefinite Kaehler and indefinite Sasakian manifolds which is an umbrella of CR-lightlike, SCR-lightlike, Screen real GCR-lightlie submanifolds. In [10], M. Atceken and E. Kilic introduced semi-invariant lightlike submanifolds of a semi-Riemannian product manifold. In [12], E. Kilic and B. Sahin introduced radical anti-invariant lightlike submanifolds of a semi-Riemannian product and gave some examples and results for lightlike submanifolds. In [13] E. Kilic and O. Bahadir studied lightlike hypersurfaces of a semi-Riemannian product manifold with respect to quarter symmetric non-metric connection.
In this paper, we study half-lightlike submanifolds of a semi-Riemannian product manifold. In Section 2, we give some basic concepts. In Section 3, we introduce screen semi-invariant half-lightlike submanifolds, screen invaryant half-lightlike submanifolds and radical anti-invariant half-lightlike submanifolds of a semi-Riemannian product manifold. We defined some special distribution of screen semi-invariant half-lightlike submanifold. We give some examples and study their geometric properties.

2010 *Mathematics Subject Classification.* 53C15, 53C25, 53C40.
Key words and phrases. Half-lightlike submanifold , Product manifolds, Screen semi-invariant.

2. Half-lightlike submanifolds

Let $(\widetilde{M}, \widetilde{g})$ be an $(m+2)$-dimensional $(m > 1)$ semi-Riemannian manifold of index $q \geq 1$ and M a submanifold of codimension 2 of $\widetilde{M}$. If $\widetilde{g}$ is degenerate on the tangent bundle TM on M, then M is called a lightlike submanifold of $\widetilde{M}$ [11]. Denote by g the induced degenerate metric tensor of $\widetilde{g}$ on M. Then there exists locally (or globally) a vector field $\xi \in \Gamma(TM)$, $\xi \neq 0$, such that $g(\xi, X) = 0$ for any $X \in \Gamma(TM)$. For any tangent space T_xM, $(x \in M)$, we consider

$$T_xM^{\perp} = \{u \in T_x\widetilde{M} : \widetilde{g}(u, v) = 0, \forall v \in T_xM\}, \tag{1}$$

a degenerate 2-dimensional orthogonal (but not complementary) subspace of $T_x\widetilde{M}$. The radical subspace $Rad\ T_xM = T_xM \cap T_xM^{\perp}$ depends on the point $x \in M$. If the mapping

$$Rad\ TM : x \in M \longrightarrow Rad\ T_xM \tag{2}$$

defines a radical distribution on M of rank $r > 0$, then the submanifold M is called $r-$lightlike submanifold. If $r = 1$, then M is called half-lightlike submanifold of $\widetilde{M}$ [11]. Then there exist $\xi, u \in T_xM^{\perp}$ such that

$$\widetilde{g}(\xi, v) = 0, \quad \widetilde{g}(u, u) \neq 0, \forall v \in T_xM^{\perp}. \tag{3}$$

Furthermore, $\xi \in Rad\ T_xM$, and

$$\widetilde{g}(\xi, X) = \widetilde{g}(\xi, v) = 0, \forall X \in \Gamma(TM),\ v \in \Gamma(TM^{\perp}). \tag{4}$$

Thus, $Rad\ TM$ is locally (or globally) spanned by ξ. By denote the complementary vector bundle $S(TM)$ of $Rad\ TM$ in TM which is called screen bundle of M. Thus we have the following decomposition

$$TM = Rad\ TM \perp S(TM), \tag{5}$$

where $\perp$ denotes the orthogonal-direct sum. In this paper, we assume that M is half-lightlike. Then there exists complementary non-degenerate distribution $S(TM^{\perp})$ of $Rad\ TM$ in $TM^{\perp}$ such that

$$TM^{\perp} = Rad\ TM \perp S(TM^{\perp}). \tag{6}$$

Choose $u \in S(TM^{\perp}$ as a unit vector field with $\widetilde{g}(u, u) = \epsilon = \pm 1$. Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\widetilde{M}$. We note that ξ and u belong to $S(TM)^{\perp}$. Thus we have

$$S(TM)^{\perp} = S(TM^{\perp}) \perp S(TM^{\perp})^{\perp},$$

where $S(TM^{\perp})^{\perp}$ is the orthogonal complementary to $S(TM^{\perp})$ in $S(TM)^{\perp}$. For any null section ξ of $Rad\ TM$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a uniquely determined null vector field $N \in \Gamma(ltr(TM))$ satisfying

$$\widetilde{g}(\xi, N) = 1,\ \widetilde{g}(N, N) = \widetilde{g}(N, X) = \widetilde{g}(N, u) = 0, \forall X \in \Gamma(TM), \tag{7}$$

where N, $ltr(TM)$ and $tr(TM) = S(TM^{\perp}) \perp ltr(TM)$ are called the lightlike transversal vector field, lightlike transversal vector bundle and transversal vector bundle of M with respect to $S(TM)$, respectively. Then we have the following decomposition:

$$T\widetilde{M} = TM \oplus tr(TM) = S(TM) \perp \{Rad\ TM \oplus ltr(TM)\} \perp S(TM^{\perp}). \tag{8}$$

Let $\widetilde{\nabla}$ be the Levi-Civita connection of $\widetilde{M}$ and P the projection of TM on $S(TM)$ with respect to the decomposition (5). Thus, for any $X \in \Gamma(TM)$, we can write $X = PX + \eta(X)\xi$, where η is a local differential 1-form on M given by $\eta(X) = \widetilde{g}(X, N)$. Then the Gauss and Weingarten formulas are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + D_1(X,Y)N + D_2(X,Y)u, \tag{9}$$
$$\widetilde{\nabla}_X U = -A_U X + \nabla^t_X U, \tag{10}$$
$$\widetilde{\nabla}_X N = -A_N X + \tau(X)N + \rho(X)u, \tag{11}$$
$$\widetilde{\nabla}_X u = -A_u X + \phi(X)N, \tag{12}$$
$$\nabla_X PY = \nabla^*_X PY + E(X, PY)\xi, \tag{13}$$
$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi, \tag{14}$$

for any $X, Y \in \Gamma(TM)$, $U \in \Gamma(tr(TM))$, where ∇, ∇^* and ∇^t are induced linear connections on M, $S(TM)$ and $tr(TM)$, respectively, D_1 and D_2 are called the local second fundamental of M, E is called the local second fundamental form on $S(TM)$. A_U, A_N, A^*_ξ and A_u are linear operators on TM and τ, ρ and ϕ are $1-$forms on TM. We note that, the induced connection ∇ is torsion-free but it is not metric connection on M and satisfies

$$(\nabla_X g)(Y,Z) = D_1(X,Y)\eta(Z) + D_1(X,Z)\eta(Y), \tag{15}$$

for any $X, Y, Z \in \Gamma(TM)$. However the connection ∇^* on $S(TM)$ is metric. From the above statements, we have

$$D_1(X,Y) = g(A^*_\xi X, Y), \quad g(A^*_\xi X, N) = 0, \quad D_1(X,\xi) = 0, \quad \widetilde{g}(A_N X, N) = 0, \tag{16}$$
$$E(X, PY) = g(A_N X, PY), \quad \epsilon D_2(X,Y) = g(A_N X, Y) - \phi(X)\eta(Y), \tag{17}$$
$$\epsilon\rho(X) = \widetilde{g}(A_u X, N), \; \tau(X) = -\eta(\nabla_X \xi), \; \rho(X) = \epsilon\eta(A_u X), \; \phi(X) = -\epsilon D_2(X,\xi), \tag{18}$$

for any $X, Y \in \Gamma(TM)$. From (16) and (17), A^*_ξ and A_N are $\Gamma(S(TM))-$valued shape operators related to D_1 and E, respectively and $A^*_\xi \xi = 0$.
For basic information on the geometry of lightlike submanifolds, we refer to [1], [11].

Let ($\widetilde{M}$ be an $n-$ dimensional diferentiable manifold with a tensor field F of type $(1,1)$ on $\widetilde{M}$ such that $F^2 = I$. Then M is called an almost product manifold with almost product structure F. If we put $\pi = \frac{1}{2}(I+F)$, $\sigma = \frac{1}{2}(I-F)$ then we have

$$\pi + \sigma = I, \; \pi^2 = \pi, \; \sigma^2 = \sigma, \; \pi\sigma = \sigma\pi = 0, \; F = \pi - \sigma.$$

Thus π and σ define two complementary distributions and the eigenvalue of F are ∓ 1. If an almost product manifold $\widetilde{M}$ admits a semi-Riemannian metric $\widetilde{g}$ such that

$$\widetilde{g}(FX, FY) = \widetilde{g}(X,Y), \; \widetilde{g}(FX, Y) = \widetilde{g}(X, FY), \forall X, Y \in \Gamma(\widetilde{M}),$$

then $(\widetilde{M}, \widetilde{g})$ is called semi-Riemannian almost product manifold. If, for any X, Y vector fields on $\widetilde{M}$, $(\widetilde{\nabla}_X F)Y = 0$, that is

$$\widetilde{\nabla}_X FY = F\widetilde{\nabla}_X Y,$$

then M is called an semi-Riemannian product manifold, where $\widetilde{\nabla}$ is the Levi-Civita connection on $\widetilde{M}$.

3. Screen Semi-Invariant Lightlike Submanifolds

Let (M,g) be a half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M},\widetilde{g})$ For any $X \in \Gamma(TM)$ we can write

$$FX = fX + wX, \tag{19}$$

where f and w are the projections on of $\Gamma(T\widetilde{M})$ onto TM and $trTM$, respectively, that is, fX and wX are tangent and transversal components of FX. From (8) and (19), we can write

$$FX = fX + w_1(X)N + w_2(X)u, \tag{20}$$

where $w_1(X) = \widetilde{g}(FX,\xi)$, $w_2(X) = \epsilon\widetilde{g}(FX,u)$.

Definition 3.1. *Let (M,g) be a half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M},\widetilde{g})$. If $FRad\,TM \subset S(TM)$, $Fltr(TM) \subset S(TM)$ and $F(S(TM^{\perp})) \subset S(TM)$ then we say that M is a screen semi-invaryant (SSI) half-lightlike submanifold.*

If $FS(TM) = S(TM)$, then we say that M is a screen invaryant half-lightlike submanifold.

Now, let M be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M},\widetilde{g})$. If we set $L_1 = FRad\,TM$, $L_2 = Fltr(TM)$ and $L_3 = F(S(TM^{\perp}))$, then we can write

$$S(TM) = L_0 \perp \{L_1 \oplus L_2\} \perp L_3, \tag{21}$$

where L_0 is a $(m-3)-$dimensional distribution. Hence we have the following decompositions:

$$TM = L_0 \perp \{L_1 \oplus L_2\} \perp L_3 \perp Rad\,TM, \tag{22}$$

$$T\widetilde{M} = L_0 \perp \{L_1 \oplus L_2\} \perp L_3 \perp S(TM^{\perp}) \perp \{Rad\,TM \oplus ltr(TM)\}. \tag{23}$$

Example 3.1. *Let M_1 and M_2 be R^4_2 and R^3_2, respectively. Then $\widetilde{M} = M_1 \times M_2$ is a semi-Riemannian product manifold with metric tensor $\widetilde{g} = \pi^* g_1 + \sigma^* g_2$ and the product structure $F = \pi_* - \sigma_*$, where g_1 and g_2 are standard metric tensors of R^4_2 and R^3_2, π_* and σ_* are the projection maps of $\Gamma(T\widetilde{M})$ onto $\Gamma(TM_1)$ and $\Gamma(TM_2)$, respectively. We consider in $\widetilde{M}$ the submanifold M given by the following equations;*

$$\begin{aligned}
x_1 &= t_1 + t_2 - t_3, \\
x_2 &= t_1 + t_2 + t_3 + \sqrt{2}\arctan t_4, \\
x_3 &= \sqrt{2}(t_1 + t_2 + t_3) + \arctan t_4, \\
x_4 &= t_5, \\
x_5 &= t_1 - t_2 + t_3, \\
x_6 &= \arctan t_4, \\
x_7 &= t_1 - t_2 - t_3,
\end{aligned}$$

where t_i are real parameters. Then we have

$$TM = Span\{U_1, U_2, U_3, U_4, U_5\},$$

where

$$\begin{aligned} U_1 &= \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \sqrt{2}\frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_5} + \frac{\partial}{\partial x_7}, \\ U_2 &= \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \sqrt{2}\frac{\partial}{\partial x_3} - \frac{\partial}{\partial x_5} - \frac{\partial}{\partial x_7}, \\ U_3 &= -\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \sqrt{2}\frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_5} - \frac{\partial}{\partial x_7}, \\ U_4 &= \frac{\sqrt{2}}{(1+t_4^2)}\frac{\partial}{\partial x_2} + \frac{1}{(1+t_4^2)}\frac{\partial}{\partial x_3} + \frac{1}{(1+t_4^2)}\frac{\partial}{\partial x_6}, \\ U_5 &= \frac{\partial}{\partial x_4}. \end{aligned}$$

We easily check that the vector U_1 is a degenerate vector, M is a $1-$ lightlike submanifold of $\widetilde{M}$. We set $\xi = U_1$, then we have $Rad\ TM = Span\{\xi\}$ and $S(TM) = Span\{U_2, U_3, U_4, U_5\}$. We can easily obtain that

$$ltr(TM) = Span\{N = -\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \sqrt{2}\frac{\partial}{\partial x_3} - \frac{\partial}{\partial x_5} + \frac{\partial}{\partial x_7}\},$$

and

$$S(TM^{\perp}) = Span\{u = \sqrt{2}\frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_6}\}.$$

Thus M is a half-lightlike submanifold of $\widetilde{M}$. Furthermore, we get

$$F\xi = U_2, \ \ FN = U_3, \ \ Fu = (1+t_4^2)U_4, \ \ FU_5 = U_5.$$

If we set $L_0 = Span\{U_5\}$, $L_1 = Span\{U_2\}$, $L_2 = Span\{U_3\}$, $L_3 = Span\{U_4\}$, then M is a screen semi-invariant half-lightlike submanifold of $\widetilde{M}$.

Example 3.2. *Consider in $\widetilde{M} = R_1^5$ the submanifold M given by the equations*

$$x_1 = x_3, \ x_5 = \sqrt{1 - \{x_2^2 + x_4^2\}}.$$

Then we have

$$TM = Span\{\xi = \partial x_1 + \partial x_3, \ Z_1 = x_5 \partial x_2 - x_2 \partial x_5, \ Z_2 = x_5 \partial x_4 - x_4 \partial x_5\}.$$

It follows that M is $1-$ lightlike. We obtain

$$N = \frac{1}{2}(-\partial x_1 + \partial x_3),$$

and

$$u = x_2 \partial x_2 + x_4 \partial x_4 + \sqrt{1 - \{x_2^2 + x_4^2\}}\ \partial x_5.$$

In where $ltr(TM) = Span\{N\}$, $Rad\ TM = Span\{\xi\}$, $S(TM^{\perp}) = Span\{u\}$ and $S(TM) = Span\{Z_1, Z_2\}$ [11].
If we set $F(x_1, x_2, x_3, x_4, x_5) = (x_1, -x_2, -x_3, -x_4, -x_5)$, then $F^2 = I$ and F is a product structure on R_1^5. Then it is easily check that, M is a screen invariant half-lightlike submanifold of $\overline{M}$.

We note that, for a screen invariant half-lightlike submanifold, it may be $FRad\ TM = Rad\ TM$, $Fltr(TM) = ltr(TM)$ and $FRad\ TM = ltr(TM)$, $Fltr(TM) = Rad\ TM$.

Example 3.3. *Let M_1 and M_2 be R_1^1 and R_1^3, respectively. Then $\widetilde{M} = M_1 \times M_2$ is a semi-Riemannian product manifold with metric tensor $\widetilde{g} = \pi^* g_1 + \sigma^* g_2$ and $F(x_1, x_2, x_3, x_4) = (x_1, x_2, -x_4, -x_3)$, where g_1 and g_2 are standart metric tensors of R_1^1, R_1^3 and (x_1, x_2, x_3, x_4) is the standard coordinate system of $R_1^1 \times R_1^3 \equiv R_2^4$, π and σ are projections on $\widetilde{M}$. We consider in $\widetilde{M}$ the submanifold M given by the equations;*

$$\begin{aligned} x_1 &= 2t_1 - 4t_2, \\ x_2 &= t_1 + 7t_2, \\ x_3 &= 2t_1 - t_2, \\ x_4 &= t_1 + t_2. \end{aligned}$$

where t_i are real parameters.
Then we have

$$TM = Span\{U_1, U_2\},$$

where

$$\begin{aligned} U_1 &= 2\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + 2\frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_4}, \\ U_2 &= -4\frac{\partial}{\partial x_1} - 7\frac{\partial}{\partial x_2} - \frac{\partial}{\partial x_3} + \frac{\partial}{\partial x_4}. \end{aligned}$$

We easily check that the vector U_1 is a degenerate vector on M and M is a $1-$ lightlike submanifold of $\widetilde{M}$. We set $\xi = U_1$, then we have $RadT\ M = Span\{\xi\}$ and $S(TM) = Span\{U_2\}$. Then by direct calculations we obtain

$$ltr(TM) = Span\{N = 2\frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} - \frac{\partial}{\partial x_3} - 2\frac{\partial}{\partial x_4}\}$$

and

$$S(TM^{\perp}) = Span\{V = \frac{\partial}{\partial x_1} + 2\frac{\partial}{\partial x_3} - 2\frac{\partial}{\partial x_4}\}.$$

Then we get $F\xi = N$. Thus M is a screen invariant half-lightlike submanifold of $\widetilde{M}$.

From the $Definition$ 3.1 and (22), we have the following proposition.

Proposition 3.1. *Let M be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold. The distribution L_0 is a invariant distribution with respect to F.*

Since F is parallel with respect to $\widetilde{\nabla}$, from (9), (10) and (19), we obtain

$$\begin{aligned} \widetilde{\nabla}_X FY &= \widetilde{\nabla}_X fY + \widetilde{\nabla}_X wY \\ &= \nabla_X fY + D_1(X, fY)N + D_2(X, fY)u - A_{wY}X + \nabla^t_X wY, \\ F\widetilde{\nabla}_X Y &= F\nabla_X Y + D_1(X, Y)FN + D_2(X, Y)Fu \\ &= f\nabla_X Y + w\nabla_X Y + D_1(X, Y)FN + D_2(X, Y)Fu, \end{aligned}$$

or

$$(\nabla_X f)Y = A_{wY}X + D_1(X, Y)FN + D_2(X, Y)Fu, \tag{24}$$
$$w\nabla_X Y = D_1(X, fY)N + D_2(X, fY)u + \nabla^t_X wY, \tag{25}$$

for any $X, Y \in \Gamma(TM)$.

Theorem 3.1. *Let (M, g) be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M}, \widetilde{g})$. The distribÃ¼tion L_0 is integrable if and only if*

$$D_i(X, FY) = D_i(Y, FX), \ i = 1, 2, \ E(X, FY) = E(Y, FX), \ X, Y \in \Gamma(L_0).$$

Proof. For any $X, Y \in \Gamma(L_0)$, from (9), (11) and (17), we obtain

$$\begin{aligned} g([X,Y], F\xi) &= g(\nabla_X Y - \nabla_Y X, F\xi) \\ &= g(\nabla_X Y, F\xi) - g(\nabla_Y X, F\xi) \\ &= D_1(X, FY) - D_1(Y, FX), \end{aligned} \tag{26}$$

$$\begin{aligned} g([X,Y], Fu) &= g(\nabla_X Y - \nabla_Y X, Fu) \\ &= g(\nabla_X Y, Fu) - g(\nabla_Y X, Fu) \\ &= D_2(X, FY) - D_2(Y, FX), \end{aligned} \tag{27}$$

and

$$\begin{aligned} g([X,Y], FN) &= g(\nabla_X Y - \nabla_Y X, FN) \\ &= g(\widetilde{\nabla}_X Y, FN) - g(\widetilde{\nabla}_Y X, FN) \\ &= -g(FY, \widetilde{\nabla}_X N) + g(FX, \widetilde{\nabla}_Y N) \\ &= g(A_N X, FY) - g(A_N Y, FX) \\ &= E(X, FY) - E(Y, FX). \end{aligned} \tag{28}$$

From (26), (27) and (28) the proof is completed. □

Let (M, g) be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M}, \widetilde{g})$. If we set

$$L = L_0 \perp L_1 \perp Rad\ TM \quad L^{\perp} = L_2 \perp L_3,$$

then we can write

$$TM = L \oplus L^{\perp}.$$

We note that the distribution L is a invariant distribution and the distribution $L^{\perp}$ is aanti-invariant distribution with respect to F on M.

Theorem 3.2. *Let (M, g) be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M}, \widetilde{g})$. Then the distribution L is integrable if and only if*

$$D_i(X, FY) = D_i(Y, FX), i = 1, 2, X, Y \in \Gamma(L).$$

Proof. For any $X, Y \in \Gamma(L)$, The distribution L is integrable if and only if there is no component L_2 and L_3 of $[X, Y]$. Then we obtain

$$\begin{aligned} g([X,Y], F\xi) &= g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, F\xi) \\ &= g(\widetilde{\nabla}_X FY - \widetilde{\nabla}_Y FX, \xi) \\ &= D_1(X, FY) - D_1(Y, FX), \end{aligned}$$

and

$$\begin{aligned} g([X,Y], Fu) &= g(\widetilde{\nabla}_X FY - \widetilde{\nabla}_Y FX, u) \\ &= D_2(X, FY) - D_2(Y, FX). \end{aligned}$$

Thus proof is completed. □

Theorem 3.3. *Let (M, g) be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M}, \widetilde{g})$. Then the distribution $L^{\perp}$ is integrable if and only if*

$$A_{FY}X = A_{FX}Y, X, Y \in \Gamma(L^{\perp}).$$

Proof. For any $X, Y \in \Gamma(L^{\perp})$, $Z \in \Gamma(L_0)$ and $N \in \Gamma(ltrTM)$. The distribution $L^{\perp}$ is integrable if and only there is no component L_0, L_1 and $RadTM$ of $[X, Y]$. Then we obtain

$$\begin{aligned} g([X,Y], FN) &= g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, FN) \\ &= g(\widetilde{\nabla}_X FY - \widetilde{\nabla}_Y FX, N) \\ &= g(A_{FX}Y - A_{FY}X, N), \end{aligned}$$

and

$$\begin{aligned} g([X,Y], FZ) &= g(\widetilde{\nabla}_X Y - \widetilde{\nabla}_Y X, FZ) \\ &= g(\widetilde{\nabla}_X FY - \widetilde{\nabla}_Y FX, Z) \\ &= g(A_{FX}Y - A_{FY}X, Z). \end{aligned}$$

Similarly, there is no component $RadTM$ of $[X, Y]$. Thus proof is completed. □

Definition 3.2. *Let (M, g) be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M}, \widetilde{g})$. Then M is mixed totally geodesic if*

$$D_i(X, Y) = 0,\ i \in \{1, 2\},$$

for any $X \in \Gamma(L^{\perp})$ and $Y \in \Gamma(L)$.

Theorem 3.4. *Let (M, g) be a screen semi-invariant half-lightlike submanifold of a semi-Riemannian product manifold $(\widetilde{M}, \widetilde{g})$. Then the following assertions are equivalent.*
(i) *M is mixed totally geodesic.*
(ii) *$A_{FY}X = -f\nabla_X Y$, $X \in \Gamma(L)$ and $Y \in \Gamma(L^{\perp})$.*
(iii) *$(\nabla_X f)Y = 0$, $X \in \Gamma(L^{\perp})$ and $Y \in \Gamma(L)$*

Proof. We know that

$$(\widetilde{\nabla}_X F)Y = 0.$$

For any $X \in \Gamma(L)$, $Y \in \Gamma(L^{\perp})$, we get

$$\begin{aligned} \widetilde{\nabla}_X FY &= F\widetilde{\nabla}_X Y, \\ -A_{FY}X + \nabla^t_X FY &= F\nabla_X Y + D_1(X,Y)FN + D_2(X,Y)Fu. \end{aligned}$$

This equation is divided into tangential and normal components, we have

$$-A_{FY}X = f\nabla_X Y + D_1(X,Y)FN + D_2(X,Y)Fu.$$

and this is $(i) \Leftrightarrow (ii)$.
For any $X \in \Gamma(L^{\perp})$, $Y \in \Gamma(L)$, from (24) and $wY = 0$, we obtain

$$(\nabla_X f)Y = D_1(X,Y)FN + D_2(X,Y)Fu.$$

Thus we have $(i) \Leftrightarrow (iii)$. □

REFERENCES

[1] Duggal, Krishan L. and Bejancu, A., *Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications*, Kluwer Academic Publishers, Dordrecht, 1996.

[2] Duggal, K. L. and Bejancu, A. *Lightlike submanifolds of codimension two*, Math. J. Toyama Univ., 15(1992), 59–82.

[3] Duggal, K.L. and Jin, D.H., *Null Curves and Hypersurfaces of Semi-Riemannian manifolds*, World Scientific Publishing Co. Pte. Ltd., 2007.

[4] Duggal, K. L. *Riemannian geometry of half lightlike submanifolds*, Math. J. Toyama Univ., 25, (2002), 169–179.

[5] Duggal, K. L. and Sahin, B. *Screen conformal half-lightlike submanifolds*, Int.. J. Math., Math. Sci., 68, (2004), 3737–3753.

[6] Duggal, K. L. and Sahin, B. *Screen Cauchy Riemann lightlike submanifolds*, Acta Math. Hungar., 106(1-2) (2005), 137–165

[7] Duggal, K. L. and Sahin, B. *Generalized Cauchy Riemann lightlike submanifolds*, Acta Math. Hungar., 112(1-2), (2006), 113–136.

[8] Duggal, K. L. and Sahin, B. *Lightlike submanifolds of indefinite Sasakian manifolds*, Int. J. Math. Math. Sci., 2007, Art ID 57585, 1–21.[162]

[9] Duggal, K. L. and Sahin, B. C*ontact generalized CR-lightlike submanifolds of Sasakian submanifolds*, Acta Math. Hungar., 122, No. 1-2, (2009), 45–58.

[10] Atceken, M. and Kilic, E., *Semi-Invariant Lightlike Submanifolds of a Semi-Riemannian Product Manifold*, Kodai Math. J., Vol. 30, No. 3, (2007), pp. 361-378.

[11] Duggal K. L., Sahin B., *Differential Geometry of Lightlike Submanifolds*, Birkhauser Veriag AG Basel-Boston-Berlin (2010).

[12] Kilic, E. and Sahin, B., *Radical Anti-Invariant Lightlike Submanifolds of a Semi-Riemannian Product Manifold*, Turkish J. Math., 32, (2008), 429-449.

[13] Kilic, E. and Bahadir, O., *Lightlike Hypersurfaces of a Semi-Riemannian Product Manifold and Quarter-Symmetric Nonmetric Connections*, Hindawi Publishing Corporation International Journal of Mathematics and Mathematical Sciences Volume 2012, Article ID 178390, 17 pages.

DEPARTMENT OF MATHEMATICS,
FACULTY OF ARTS AND SCIENCES, K.S.U.
KAHRAMANMARAS, TURKEY
E-mail address: oguzbaha@gmail.com