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ROTATING BLACK HOLE AND A POTENTIAL FOR ITS WEYL TENSOR

LAURIAN-IOAN PIŞCORAN, J. LÓPEZ-BONILLA, AND G. PÉREZ-TERUEL

ABSTRACT. We show that with a change of sign into two strategic components of the metric tensor in Kerr geometry, a generator for the Lanczos potential of this rotating black hole, can be obtained.

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1. INTRODUCTION

Any physical theory in a Riemannian 4-space (as in general relativity) should include the dynamical participation of the Lanczos potential, and to elucidate its meaning into such theory. Actually the Einsteinian gravitation not incorporates in an active form the Lanczos generator, and it is an open question its possible physical meaning. In our opinion the Lanczos spintensor is an angular momentum density, for example, it could have connection with the rotation of a black hole, idea that can be investigated if first a Lanczos potential is determined explicitly for the Kerr metric, the present analysis shows how to obtain it.

Lanczos, 1962 showed, for any spacetime, the existence of a potential K_{abc} with the properties:

$$K_{abc} = -K_{bac}; \quad K_{abc} + K_{bca} + K_{cab} = 0 \tag{1.1}$$

which generates the conformal tensor through the following relation, Bampi and Caviglia, 1983, Roberts, 1988 and 1992, Edgar, 1994, Agacy and Briggs, 1994, Edgar and Höglund, 1997 and 2000:

$$C_{abcd} = K_{abc;d} - K_{abd;c} + K_{cda;b} + \frac{1}{2}[g_{ad}\left(K_{bc} + K_{cb}\right) - g_{ac}\left(K_{bd} + K_{db}\right) + g_{bc}\left(K_{ad} + K_{da}\right)$$
$$- g_{bd}\left(K_{ac} + K_{ca}\right)] + \frac{2}{3}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)K^{pq}_{p;q} \tag{1.2}$$

such that:

$$K_{ij} = K_{i\ j;p}^{p} - K_{i\ p;j}^{p} \tag{1.3}$$

Given the Weyl tensor, it may be a formidable task to construct a Lanczos generator by integrating directly the system (1.2) of differential equations for the unknown K_{abc} . However, here we exhibit that the Lanczos potential for the important geometry of Kerr, 1963, can be generated with remarkable simplicity. In fact, we consider the metric of a rotating black hole, Chandrasekhar, 1983, in the Boyer and Lindquist, 1967 coordinates $(r, \theta, \vartheta, t)$ with signature +2:

$$ds^2 = \frac{\Sigma}{C}dr^2 + \Sigma d\theta^2 - \frac{4amr\sin^2\theta}{\Sigma}d\vartheta dt + \sin^2\theta\left(r^2 + a^2 + \frac{2a^2mr\sin^2\theta}{\Sigma}\right)d\vartheta^2 - \left(1 - \frac{2mr}{\Sigma}\right)dt^2 \tag{1.4}$$

where $\Sigma = r^2 + a^2\cos^2\theta$, $C = r^2 - 2mr + a^2$ and besides, we introduce the following symmetric tensor:

$$(S_{ij}) = \begin{pmatrix} \epsilon g_{11} & 0 & 0 & 0 \\ 0 & \epsilon g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & g_{34} \\ 0 & 0 & g_{34} & g_{44} \end{pmatrix}, \tag{1.5}$$

being g_{ab} the metric tensor corresponding to (1.4). If into (1.5), we put $\epsilon = 1$ then $S_{ij} = g_{ij}$ with the known property $g_{ab;c} = 0$. But if we take $\epsilon = -1$, then (1.5) resolves (1.2) because it permits to construct a Lanczos potential for the Kerr spacetime via the expression:

$$K_{ijc} = \frac{1}{4}\left(S_{cj;i} - S_{ci;j}\right) \tag{1.6}$$

It is a true surprise that a simple change of sign in ϵ gives a solution for the complicated system (1.2). An open problem is to find the underlying principle in this result for the black hole with rotation.

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TECHNICAL UNIVERSITY OF CLUJ NAPOCA
NORTH UNIVERSITY CENTER OF BAIA MARE
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
VICTORIEI 76, 430122 BAIA MARE, ROMANIA
E-mail address: plaurian@yahoo.com

2 ESIME-ZACATENCO, INSTITUTO POLITÉCNICO NACIONAL (IPN),
EDIF. 5, COL. LINDAVISTA,
CP 07738, MÉXICO DF, MEXICO
E-mail address: jlopezb@ipn.mx

DEPARTAMENTO DE FISICA TÉORICA,
UNIVERSIDAD DE VALENCIA, BURJASSOT 46 100,
VALENCIA, SPAIN