Global Journal of Advanced Research
on Classical and Modern Geometries
ISSN: 2284-5569, Vol.3, (2014), Issue 1, pp.7-11

POSITION VECTOR OF SOME SPECIAL CURVES IN GALILEAN 3-SPACE G^3

SÜHA YILMAZ, ÜMIT ZIYA SAVCI, AND ABDULLAH MAĞDEN

ABSTRACT. In this paper, position vectors of curves in Galilean space G^3 according to standart frame are investigated. Second, it is proven that position satisfies a vector differantial equation of second and fourth order. First, a system of differantial equation whose solution gives the components on the Frenet axis in G^3 is established by means of Frenet equations. Moreover, a characterizations for position vector of rectifying curves are expressed in G^3.

MSC 2010: 53A05, 53B25, 53B30.
Keywords: Frénet frame, Galilean space, position vector, rectifying curves.

1. INTRODUCTION

In the Euclidean 3-space, rectifying curves are introduced by B.Y. Chen in [2] as space curves whose position vector always lies in its rectifying plane, spanned by the tangent and the binormal vector fields T and B of the curve. Accordingly, the position vector with respect to some chosen origin, of a rectifying curve α in E^3, satisfies the equation

$$\alpha(s) = \lambda(s)T(s) + \mu(s)B(s) \tag{1}$$

where $\lambda(s)$ and $\mu(s)$ are arbitrary differential functions in arc-length parameter $s \in I \subset \mathbb{R}$.

In the Minkowski 3-space, position vector of some special space-like curves according to Bishop frame are studied by S. Yılmaz in [6]. Morever, A. Ali investigated to position vector of a time-like slant helix in Minkowski 3-space in [1] . Apart from, S. Yılmaz studied position vector of a partially null curve derived from a vector equation [7] .

In this paper, in analogy with Euclidean 3-dimensional case, we define the rectifying curve in the Galilean space G^3.

2. PRELIMINARIES

The Galilean space is a three dimensional complex projective space, P^3, in which the absolute figure $\{w, f, I_1, I_2\}$ consist of a real plane w (the absolute plane) a real line $f \subset w$ (the absolute line) and two complex conjugate points $I_1, I_2 \in f$ (the absolute points) [4]

We shall take, as a real model of the space G^3, a real projective space P^3, with the absolute $\{w, f, \}$ consist of a real plane $w \subset G^3$ and a real line $f \subset w$, on which an elliptic involution ε has been defined.

Let ε be homogeneous coordinates

$$\begin{array}{ll} w...x_{0,} & f...x_0 = x_1 = 0 \\ \varepsilon : (0:0: & x_2 : x_3) \longrightarrow (0:0:x_3:-x_2) \end{array}$$

The scalar product Galilean space G^3 is defined by

$$< X, Y >_{G^3} = \begin{cases} x_1 y_1 & , \text{ if } x_1 \neq 0 \text{ or } y_1 \neq 0 \\ x_2 y_2 + x_3 y_3 & , \text{ if } x_1 = 0 \text{ and } y_1 = 0 \end{cases}$$

The cross product Galilean space G^3 is defined by

$$X \wedge Y = \begin{vmatrix} 0 & e_1 & e_2 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} \tag{2}$$

Where $X = (x_1, x_2, x_3)$, $Y = (y_1, y_2, y_3)$.
Let $\alpha : I \longrightarrow G^3$, $I \subset \mathbb{R}$ be a curve in Galilean space G^3 given by $\alpha = (x, y(x), z(x))$ where x is a Galilean invariant-the arc lenght on α,

$$\kappa(x) = \sqrt{(y''(x))^2 + (z''(x))^2}, \ \ \tau(x) = \frac{\det(\alpha'(x), \alpha''(x), \alpha'''(x))}{\kappa^2(x)} \tag{3}$$

respectively,
The orthonormal trihedron of the curves α is defined by

$$\begin{aligned} \alpha'(x) = \ \ T(x) &= \ (1, y'(x), \ z'(x)) \\ N(x) &= \ \frac{1}{\kappa(x)}(0, \ y''(x), \ z''(x)) \\ B(x) &= \ \frac{1}{\kappa(x)}(0, \ -z''(x), \ y''(x)) \end{aligned} \tag{4}$$

The vectors $T(x), N(x), B(x)$ are called the tangent, principal normal and the binormal line, respectively. For their derivatives, the following Frenet formulas hold [9] .

$$\begin{aligned} T'(x) &= \kappa(x) N(x) \\ N'(x) &= \tau(x) B(x) \\ B'(x) &= -\tau(x) N(x) \end{aligned} \tag{5}$$

3. Position Vector of Regular Curves in G^3

Let $\varphi = \varphi(s)$ be unit speed curve in G^3. We can write this position vector with respect to Frenet frame in G^3 as

$$\varphi = \varphi(s) = m_1 T + m_2 N + m_3 B \tag{6}$$

differentiating (6) and considering Frenet equations, we have a system of differential equation as follow

$$\begin{aligned} & m_1' - 1 = 0 \\ & m_2' + m_1 \kappa - m_3 \tau = 0 \\ & m_2 \tau + m_3' = 0 \end{aligned} \tag{7}$$

this system's general solution have not been found. Due to this, we give some special values to the components.

Case 1: From $(7)_1$ $m_1^{'} - 1 = 0 \Rightarrow m_1^{'} = 1$. In this case, we obtain $m_1 = s + c_1$, from $(7)_1$ and $(7)_2$ gives third component as:

$$m_3 = \frac{1}{\tau} m_2^{'} + (s + c_1)\left(\frac{\kappa}{\tau}\right) \tag{8}$$

and differentiating (8) and using formulas (7) we obtain

$$m_2^{''} - \frac{\tau}{\tau^{'}} m_2^{'} + \tau^2 m_2 + \kappa + \tau \left(\frac{\kappa}{\tau}\right)^{'} (s + c_1) = 0 \tag{9}$$

And here, let us suppose $\kappa = 0$ and τ costant. Using these in the system of differential equation, we have second order homogenous differential equation with constant coefficient as follow:

$$m_2^{''} + \tau^2 m_2 = 0 \tag{10}$$

Solution of (10) is elemantary. Then, we write the components

$$m_2 = A_1 \cos(\tau s) + A_2 \sin(\tau s) \tag{11}$$

Case 2: Let us suppose $m_2 = c_2 \neq 0$ and costant. Thus, from $(7)_3$ we obtain

$$c_2 = -\frac{m_3^{'}}{\tau} \tag{12}$$

From $(7)_2$, we write

$$m_3 = m_1 \frac{\kappa}{\tau} \tag{13}$$

In this case, if (12) and (13) are used in $(7)_3$, we have a differential equation as follow

$$-m_3^{'} + \frac{\kappa}{\tau} + (s + c_1)\left(\frac{\kappa}{\tau}\right)^{'} = 0 \tag{14}$$

From solution of (14), using (12) and (13) we have the components

$$\begin{aligned} m_1 &= s + c_1 \\ m_3 &= (s + c_1)(\frac{\kappa}{\tau}) \\ \frac{\kappa}{\tau} &= \frac{-c_2 \int \tau ds}{s + c_1} \end{aligned} \tag{15}$$

where c_1 and $c_2 \in \mathbb{R}$

Case 3: $m_3 = 0$. Thus $\tau = 0$, and here we obtain

$$\begin{gathered} m_2 = -(s + c_1)\kappa ds \\ m_1 = s + c_1 \\ m_3 = 0 \end{gathered}$$

Considering $(15)_1$ and $(15)_2$, we give following theorem.

Theorem 3.1: Let $\varphi = \varphi(s)$ be a unit speed rectifying curve in G^3 then;

i) There is a relation among curvatures as $(s + c_1)(\frac{\kappa}{\tau})$

ii) Position vector of φ can be written as follow

$$\varphi = \varphi(s) = (s + c_1)T + (s + c_1)(\frac{\kappa}{\tau})B$$

where $c_1 \in \mathbb{R}$

.**Proof:** Let $\varphi = \varphi(s)$ be a unit speed rectifying curve in G^3, with non-zero curvatures $\kappa(s)$ and $\tau(s)$. By definition, the position vector of the curve φ satisfies the equation (1), for some differentiable functions $\lambda(s)$ and $\mu(s)$. Differentiating the equation (1) with respect to s and using the Frenet equation (5), we obtain

$$T = \lambda' T + \mu' B$$

it follow that

$$\begin{aligned} \lambda' &= 1 \\ \mu' &= 0 \end{aligned}$$

and therefore

$$\begin{aligned} \lambda(s) &= s + a_1 \\ \mu(s) &= \frac{\kappa}{\tau}(s + a_1) = a_2 \end{aligned}$$

and here we get

$$\frac{\kappa}{\tau} = \frac{a_2}{s + a_1}$$

where a_1 and $a_2 \in \mathbb{R}$.

Theorem 3.2: Let $\eta : I \in \mathbb{R} \to G^3$ be an unit speed regular curve with curvatures $\kappa \neq 0$ and $\tau \neq 0$ in G^3. Position vector and curvatures of satisfies a vector differential equation of fourth order.

Proof: Let $\eta : I \in \mathbb{R} \to G^3$ be an unit speed regular curve with curvatures $\kappa \neq 0$ and $\tau \neq 0$ in G^3. Considering Frenet equations, we write that

$$\begin{aligned} N &= \frac{T'}{\kappa} \\ B &= \frac{N'}{\tau} \end{aligned} \tag{16}$$

Substituting $(16)_1$ in $(5)_3$, we get

$$B' = -\frac{\tau}{\kappa} T' \tag{17}$$

Then differentiating $(16)_1$ and substituting $(16)_2$, we find

$$B = \frac{1}{\tau} \left[\left(\frac{1}{\kappa} \right)' T' + \left(\frac{1}{\kappa} \right) T'' \right] \tag{18}$$

similarly taking the differentiating (18) and equalize it with $(5)_3$ we obtain

$$\begin{aligned} &\frac{1}{\tau} \cdot \frac{1}{\kappa} T''' + \left[2\frac{1}{\tau} \left(\frac{1}{\kappa} \right)' + \left(\frac{1}{\tau} \right)' \frac{1}{\kappa} \right] T'' + \\ &\left[\frac{1}{\tau} \left(\left(\frac{1}{\kappa} \right)'' + \frac{\tau^2}{\kappa} \right) + \left(\frac{1}{\tau} \right)' \left(\frac{1}{\kappa} \right)' \right] T' = 0 \end{aligned} \tag{19}$$

Substituting $\eta' = T$, $\eta'' = T'$, $\eta''' = T''$, $\eta^{(IV)} = T'''$, in (19) we obtain

$$\frac{1}{\tau\kappa}\eta^{(IV)}+\left[2\frac{1}{\tau}\left(\frac{1}{\kappa}\right)'+\left(\frac{1}{\tau}\right)'\frac{1}{\kappa}\right]\eta'''+ \left[\frac{1}{\tau}\left(\left(\frac{1}{\kappa}\right)''+\frac{\tau^2}{\kappa}\right)+\left(\frac{1}{\tau}\right)'\left(\frac{1}{\kappa}\right)'\right]\eta''=0 \tag{20}$$

The formula (20) proves the theorem as desired.

References

[1] Ali, A., Turgut M., *Position Vector of A Time-like Slant Helix in Minkowski 3-Space*, J., Math. Anal.Appl. Vol 365, 559-569, (2010).

[2] Chen, B. Y., *When Does the Position Vector of A Space Curve Always Lie in Its Rectifying Plane?*, America. Math, Mountly 110, 147-152, (2003).

[3] Divjak B., *Curves in Pseudo-Galilean Geometry*, Annales Univ. Budapest, 41, 117-128 (1998).

[4] Kamenarıvic, I., *Existence Theorems for Ruled Surfaces In The Galilean Space G^3*., Rad Hazu Math, Vol. 456(10), 183-196, (1991).

[5] Yaglom, M. I., *A Simple Non-Eucliden Geometry and Its Physical*, Springer Verlag Inc. Vol 203, (1979).

[6] Yılmaz, S., *Position Vector of Some Special Space-like Curve According to Bishop Frame in Minkowski Space*, Scientia Magna, Vol 5(1),51-55. (2009).

[7] Yılmaz, S., Özyılmaz, E., Turgut, M., Nizamoğlu, Ş., *Position Vector of A Partially Null Curve Derived From A Vector Differential Equation*, International Journal of Computational and Mathematical Sciences, Vol 3(2), 71-73, (2009).

[8] Yılmaz, S., Nizamoğlu, Ş., Turgut, M., *A Note on The Differential Goemetry of The Curves in E^4*, International Journal of Mathematical Combinatorics, Vol 2, 104-108, (2008).

[9] Pavkovic B.J., Kamenaroviç I., *The Equiform Differantial Geometry of Curves In The Galilean Space G^3*, Glasnik Matematicki, Vol 22(42), 449-457, (1987).

DOKUZ EYLÜL UNIVERSITY, BUCA EDUCATIONAL FACULTY,
BUCA-IZMIR, TURKEY.
E-mail address: suha.yilmaz@deu.edu.tr

KARABÜK UNIVERSITY,
DEPARTMENT OF MATHEMATICS,78050, KARABÜK/TURKEY
E-mail address: ziyasavci@hotmail.com

ATATÜRK UNIVERSITY,
DEPARTMENT OF MATHEMATICS, ERZURUM, TURKEY
E-mail address: amagden@atauni.edu.tr