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PROBLEMS IN TRAPEZOID GEOMETRY

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ABSTRACT. The purpose of this paper is to present some known and new properties of trapezoids.

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1. INTRODUCTION

In this section, we recall the well known results:

Theorem 1.1. *(see [4] or [5]) Let a, b, c, d, be strictly positive real numbers. These numbers can be the lengths of the sides of a quadrilateral if and only if*

$$a < b + c + d, \ b < c + d + a, \ c < d + a + b, \ d < a + b + c. \tag{1.1}$$

In general, the strictly positive real numbers a, b, c, d, which verify (1.1) don't determine in a unique way a quadrilateral. We consider a quadrilateral with rigid sides and constant side lengths, its vertices being mobile articulations. Then, this quadrilateral can be deformed in order to obtain another quadrilateral. For trapezoids, the following theorem takes place:

Theorem 1.2. *(see [5]) Let a, b, c, d, be strictly positive real numbers. Then a, b, c, d can be the lengths of the sides of a trapezoid of bases a and c if and only if*

$$\begin{cases} a + d < b + c \\ a + b < c + d \\ c < a + b + d \end{cases} \quad or \quad \begin{cases} c + d < a + b \\ c + b < a + d \\ a < b + c + d \end{cases} \tag{1.2}$$

By construction, we prove that, in the condition of Theorem 1.2, the trapezoid is uniquely determined. We consider that $a < c, AD = d, DU = c - a$ and $AU = b$. The triangle ADU is uniquely determined. Let $DC = c$ and we construct the sides $AD||DC, BC||AU$. So, we obtain a trapezoid ABCD uniquely determined by the side lenghts $AB = a, BC = b, CD = c$ and $DA = d$.

2. Main Results

In this section, we consider the trapezoid ABCD, with the bases AB and CD, $AB < CD, E \in AD, EF||AB, F \in BC, EF \cap BD = \{G\}, EF \cap AC = \{H\}$ and $AC \cap BD = \{O\}$.

Theorem 2.1. *The following identities*

$$EH \equiv FG \tag{2.1}$$

and

$$EG \equiv FH \tag{2.2}$$

hold.

Proof. Because $AB||EF||CD$ (Figure 2.1), we have:

$$\frac{EA}{DA} = \frac{FB}{BC}. \tag{2.3}$$

From $EF||DC$ results that $\Delta AEH \sim \Delta ADC$ and $\Delta BFG \sim \Delta BCD$, so

$$\frac{EA}{DA} = \frac{EH}{DC} = \frac{AH}{AC} \tag{2.4}$$

and

$$\frac{FB}{BC} = \frac{FG}{CD} = \frac{BG}{BD}. \tag{2.5}$$

From (2.3)-(2.5), results (2.1) and from (2.1), relation (2.2) follows. ☐

Theorem 2.2. *We have:*

$$GH = \frac{1}{\frac{1}{AB} - \frac{1}{DC}} \left| \left(\frac{1}{AB} + \frac{1}{DC} \right) EF - 2 \right|, \tag{2.6}$$

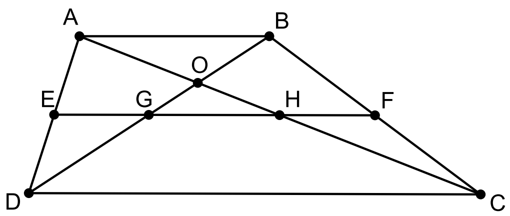

Figure 2.1

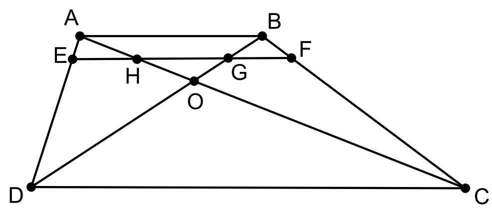

Figure 2.2

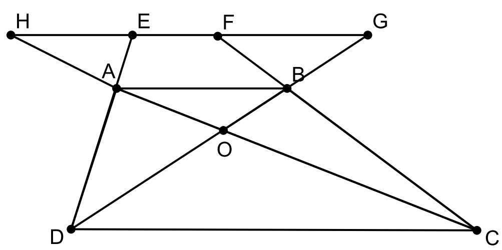

Figure 2.3

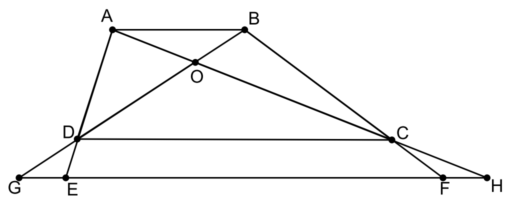

Figure 2.4

From $EF||AB$ results that $\Delta CHF \sim \Delta CAB$, from where

$$\frac{HC}{AC} = \frac{HF}{AB}. \tag{2.7}$$

From (2.4) and (2.7), we have that

$$\frac{EH}{DC} + \frac{HF}{AB} = 1 \tag{2.8}$$

if $E \in [AD]$ (see Figure 2.1),

$$\frac{HF}{AB} - \frac{EH}{DC} = 1 \tag{2.9}$$

if $E \in (DA - (AD)$ (see Figure 2.3) and

$$\frac{EH}{DC} - \frac{HF}{AB} = 1 \tag{2.10}$$

if $E \in (AD - (AD)$ (see Figure 2.4).
Taking (2.1) into account we have that $EG = FH = \frac{EF-GH}{2}$, $EH = EG + GH$ if $G \in [DO]$ (see Figure 2.1) and
$EG = FH = \frac{EF+GH}{2}$, $EH = EG - GH$ if $G \in [OB]$ (see Figure 2.2). Replacing in (2.7), one obtains:

$$GH = \frac{1}{\frac{1}{AB} - \frac{1}{DC}} \left(\left(\frac{1}{AB} + \frac{1}{DC} \right) EF - 2 \right), \tag{2.11}$$

if $G \in [DO]$ and

$$GH = -\frac{1}{\frac{1}{AB} - \frac{1}{DC}} \left(\left(\frac{1}{AB} + \frac{1}{DC} \right) EF - 2 \right), \tag{2.12}$$

if $G \in [OB]$.
Similary, we obtain that

$$GH = -\frac{1}{\frac{1}{AB} - \frac{1}{DC}} \left(\left(\frac{1}{AB} + \frac{1}{DC} \right) EF - 2 \right), \tag{2.13}$$

if $G \in (DA - (DA)$ and

$$GH = \frac{1}{\frac{1}{AB} - \frac{1}{DC}} \left(\left(\frac{1}{AB} + \frac{1}{DC} \right) EF - 2 \right), \tag{2.14}$$

if $G \in (AD - (AD)$. From (2.11)-(2.14), relation (2.6) follows.

Corollary 2.3. *If $O \in EF$, then*

$$OE \equiv OF \tag{2.15}$$

and

$$EF = \frac{2}{\frac{1}{AB} + \frac{1}{DC}}. \tag{2.16}$$

Proof. These identities result immediately from Theorem 2.1 and Theorem 2.2 in the case that G and H coincide to O. □

Corollary 2.4. *We have that*

$$EF \geq \frac{2}{\frac{1}{AB} + \frac{1}{DC}}, \tag{2.17}$$

if $G \in [DO] \cup ((AD - (AD))$ *and*

$$EF \leq \frac{2}{\frac{1}{AB} + \frac{1}{DC}}. \tag{2.18}$$

if $G \in [OB] \cup ((DA - (DA))$.

Proof. The inequalities (2.17) and (2.18), result immediately from (2.11)-(2.14). □

Corollary 2.5. *If EF is the middle line of the trapezoid, then*

$$GH = \frac{DC - AB}{2}. \tag{2.19}$$

Proof. Because $EF = \frac{DC+AB}{2}$ and replacing in (2.6), (2.19) follows. □

Corollary 2.6. *We have that* $GH = \frac{DC-AB}{2}$ *if and only if*

$$EF = \frac{DC + AB}{2} \tag{2.20}$$

for $G \in [DO] \cup ((AD - (AD))$ *or*

$$EF = \frac{6 \cdot DC \cdot AB - DC^2 - AB^2}{2(DC + AB)} \tag{2.21}$$

for $G \in [OB] - ((DA - (DA))$.

Proof. For proof, we take (2.6),(2.17) and (2.18) into account. □

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